THE

INTERACTION

OF

Wheel and Rail

AND ITS EFFECT

ON

The Motion and the Resistance of Vehicles in Trains.

From the German of

BOEDECKER:

KÖNIGHIL. EISENBAHN-BAU-UND BETRIEBS-INSPECTOR

1898.

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Ed. HEUSINGER von WALDEGG.

(" Handb. f. Spec. Eisenbahn - technik" : 4 Auf. 1877 : p. 53.)

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" Von den auf diesem Gebiete vorliegenden theoretischen Abhandlungen sei hier " nur die hervorragende Arbeit Boedecker's—' Die Wirkungen zwischen Rad und Schiene '... " erwähnt, welche manche wichtige Frage in zutreffender Weise behandelt und wesentlich " zur Klarung des Gegenstandes beigetragen hat."

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"Die werthwollen theoretischen Untersuchungen Bœdeker's."

W. LAUNHARDT,

("Theorie des Trassirens." 1888. Heft II: p. 87.)

"Bœdecker, 'Rad und Schiene'—das einzige uns bekante Werk welches über die "hier in Betracht kommenden Kräfte und Wirkungen zuverlässig Aufschluss giebt."

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REVIEW.

Die Wirkungen zwischen Rad und Schiene

und ihre Einflüsse auf den Lauf und den Bewegungswiderstand der Fahrzeuge in den Eisnbanzügen. Von **Boedecker**, Esenbahninspector zu Hannover. Hahn'sche Buchhandlung. 1887. Preis 4 M.

The above *brochure* is remarkable amongst all other hitherto published works dealing with this subject in that the various phenomena which determime the motion of Axles and Vehicles in the Track are extremely clearly investigated, and that without any of the *assumptions hitherto customary* as to the position of the vehicle, &c.

The Author's method of investigation, associated as it is with neat and telling conclusions, yields an abundance of results some of which are new, some even surprising, and many practically very valuable. In order to economise space and not to weary the busy reader the Author omits all intermediate steps in his investigations—thus ensuring that the final impression is always clear and vivid.

Chapter I treats of the motion of single axles, the distribution of pressure over the surface of contact, and of rolling-friction. To this follows, as Chapter II, the investigation of the motion of 4-wheeled vehicles in Curves.

This Chapter II is the most important of the whole work, because it contains the investigation of the forces acting at the individual wheels which is fundamental for all that follows. This investigation is distinguished from all previous attempts in this: that in determining the motion of the H(ind). A(xle). there are none of the usual assumptions—mostly erroneous—made, but the three possible vehicle positions depending on the situation of the Rotation Axis are discussed, and the actual motion is deduced solely from a consideration of the action of the forces which arise when sliding of all 4 wheels on the rails takes place.

Here the Author points out that the flange-clearance so supplements the curve-radius that the product of the two becomes *a criterion for determining the position assumed by the vehicle.* The succeeding discussion of the position of the point or surface of contact of the leading fore-wheel and the rail, when the flange-hollow of the tire is inclined at a certain angle to the inner rounding-off of the rail-head, affords opportunity for a very remarkable calculation of the curve-resistance, which clearly brings out the influence of the wheel-base, track-clearance, &c., and of which the numerical results almost exactly tally with those of von Röckl.

An investigation of the influence of the shape of flange-hollow and of rail-head on resistance and wear forms the end of this Chapter; and the conclusion is drawn that the best shapes of both have been attained with the rounding-off hitherto employed; whereas the profiles adopted in the U. S. America, in which the slide-surface of the flange is inclined to the rail at an angle of 60° with the horizontal, greatly increases the resistance, and also the tendency to sharp flanges.

And since, as appears from the investigation, some 80 % of the whole resistance occurs at the leading fore-wheel the Author calls particular attention to the importance of *lubricating the flange* of this wheel, particularly in *locomotives.*

In Chapter III, the motion of 3-axled locomotives in curves is discussed; and by means of the formulæ, previously established, for a Standard Loco. of the Prussian State Railways running without load the forces acting at the wheels and the contact-angle at the outer fore-wheel are determined.

These forces, as also the horizontal forces tending to overturn the rail, are shown to diminish comparatively rapidly in curves of large radius; from which is manifest the advantage of having the radii of curves always as large as possible.

The investigation following of a hauling 3-coupled Locomotive treats of the influence of the tractive force on the action of the previously determined forces at the wheels, and the decrease of the rail-friction through the lateral slide of the F(ore). or M(id). A(xle).

It is thus shown that, the F. A. contributes to the tractive force in curves only to a comparatively small extent; and this is further explained and illustrated by Figures.

And further, that with the decrease of the lateral force on that axle, the safety of guidance of the locomotive decreases. Consequently in Express Locomotives, the F. A. ought not to be coupled.

After this follows determinations of the curve-resistances of a 3-coupled Locomotive, and of the alteration in the wheel-loads due to the side-forces (negligible in vehicles), and to abnormal superelevation of the outer-rail. The conclusion reached is that the greatest side-pressure occurs either at the outer fore-wheel, or at the inner middle wheel, depending on the velocity. The action of the forces acting at the several wheels on the rails is illustrated for a Standard Goods Locomotive of the Prussian State Railways.

Chapter IV is a very interesting and thorough investigation of the motion of a 4-wheeled vehicle in curves of large radius and in straights; and the origin and behaviour of the serpentine, sinuous or meandering motion of vehicles, and the influence thereon of the type of vehicle—and, in particular of movable axles—is clearly explained with great skill. Some notes and data regarding the wear of rails bring the work to a conclusion.

By the investigations collected in the present book the Author has not only very clearly stated a number of disputed questions—as, for instance, the use of cylindric treads—which have not, with any certainty, been yet solved by experiment, but has likewise greatly facilitated to the thoughtful observer the task of discovering the meaning and inter-connexion of the data already extant on this subject; and has laid down the lines for future investigations; as for example, regarding the use of self-radializing trailing or hind-axles, and of Locomotive trucks: and finally, he has produced for commencing Railway Engineers *a most valuable text-book which we cannot too warmly recommend.*

v.(on) B.(orries)

In the "**Organ für die Fortschritte des Eisenbahnwesens**".

AUTHOR'S PREFACE.

MY REASON for offering to my fellow-professionals a series of discussions on the phenomena which occur between Rail and Wheel is that I attribute a great importance to this subject, and consider that its scientific treatment has, of late years, in comparison with many others in Railway Technics, been much *neglected.* The importance of the subject lies not only in its intimate connection with the question of the safety of Railway working but also in the existence of the great financial burdens which the maintenance in good working condition of the track and of wheel-tires imposes on Railway Administrations.

As this book is designed for practical use it contains only discussions which are of direct value to the working Engineer, or which are necessary to a clear understanding of the subject as a whole. On the other hand, all details of construction of the permanent-way and rolling-stock are noted so far as these enter into the question. For the rest, in the development of the theory simple and generally well-known principles have been made use of; and as to the particular methods employed, I have borrowed one from a work of mine published in the year 1873 in the "Zeitschrift für Bauwesen" on "The Motion of 4-wheeled Railway Vehicles in Curves"; and have in other cases employed new ones.

Difficulties were at first encountered in correctly expressing the influence of Flange-clearance on the motion of vehicles in curves. These were got rid of by the introduction of the device of the product of Clearance and Curve-Radius. In this way it became possible to construct comparatively simple and general expressions for the action of the horizontal forces between Rail and Wheel; and so to discuss the resulting phenomena in a clear and simple manner.

The difficulties arising also from the introduction into the discussion of the ordinary form of wheel-tread have been overcome in a similar manner.

However, owing to the complexity of the subject, it has not been possible to display the general results of the investigation in a form sufficiently simple for immediate comprehension; and for this reason they have been illustrated, numerically, by actual practical examples in which the standard types of rolling-stock in use on the Prussian State Railways have been made use of. Finally, I would remark that all tedious intermediate steps in the argument, so far as has been possible without prejudice to clearness, have been omitted.

It is the Author's hope that this little Work may contribute to increase the knowledge of the action occurring between Wheel and Rail, and to a recognition of the influence of the various individual parts of the vehicle and of the track on this action; and also that it may, perhaps, induce others to undertake further investigations in this subject.

HANNOVER. BŒDECKER.
January, 1887.

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Fig. 1.

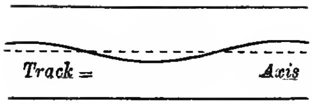

Fig. 2.

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INTRODUCTION.

When a railway vehicle runs in the usual manner on the rails its axis changes its direction in each instant relatively to the axis of the track; that is, it does not remain in the middle of the track, but describes a sinuous line.

This phenomenon, a fundamental consequence of the mode of construction of the vehicle, is much more pronounced in the straight than in curves; this is due to the fact that in passing through a curve there are certain horizontal forces acting between wheel and rail which influence the motion of the vehicle in the horizontal plane.

The axis of the wavey line described by the vehicle coincides in straights with the centre-line of the track, but in curves it always lies at some distance from it; for instance, in sharp curves it lies on the concave side. These two cases are represented in **Figs. 1** and **2.** The motion of the vehicle is perturbed and rendered momentarily irregular by defects in the build of the vehicle, and by forces arising from inequalities or defects in the track, or from other chance circumstances, which either strengthen or weaken the wave-motion.

Such perturbations also occur in straights and in curves when the wheel-flange-clearance is less than the wave-height or amplitude of the uniform swing or meander of the vehicle, the Fore Axle (= F. A.) impinging alternately against the inner and outer rail.

This uniform wave-motion takes place in very flat curves; but in such curves the centrifugal force is of little or no importance as regards vehicles and track, since the forces it produces between wheel and rail are quite insignificant. But when the uniform movement of the vehicle is interrupted by impacts of the wheel against the rail, then forces arise which have a very considerable influence thereon.

These forces act mainly in the vertical plane through the wheel's axis, and may be resolved into components perpendicular and parallel to the axis of the impinging wheel.

The vehicle is protected against the vertical blows by the springs, acting vertically, interposed between the axle-boxes and the under-frame: it is not, however, protected against the horizontal forces. These are transmitted through the axle-horns in almost undiminished intensity to the under-frame of the vehicle, and are only indirectly and partially extinguished in the bearing-springs when and only when the body of the vehicle is inclined to the direction of the blow.

This only indirectly beneficial action of the bearing-springs with respect to lateral blows and shocks is less the more nearly the position of the C. G. of the mass of the vehicle resting on the springs coincides with the axis of the impinging wheel. Were there complete coincidence the bearing-springs would have no effect whatever in diminishing the effect of the lateral blows on the car-body.

The effect of horizontal impacts on the vehicle is moderated by the circumstance that the rails yield laterally under the influence of the blow; and also, by the mounting of the impinging wheel on the rail, if the flange-hollow is sufficiently flat to allow of it. By this mountiug of the impinging wheel, a part of the horizontal blow is directed into the vertical plane and transmitted directly to the springs.

The less the rails yield laterally, and the more the mounting of the wheel is hindered by a sharp rounding-off of the flange-hollow, the more severely will a lateral or transverse blow make itself felt in the vehicle. In tracks with wooden cross-sleepers, where the rails, owing to the compressibility of the wood sleeper, have a certain amount of movement, *i.e.* rotation about a longitudinal axis lying in the foot of the rail, and when the foot of the rail itself yields laterally in proportion to the lateral bending or displacement of the outer spike,

the horizontal blows are not so severe as when the road is a metallic one; this is especially the case with iron cross-sleepers in which the rails are rigidly fixed in their normal position.

The elastic displacement of the rail-head occurring in a wooden cross-sleeper road is not by any means insignificant. Thus M. M. von Weber found that with well-maintained sleepers in a falling straight there occurred a maximum temporary increase of gauge of 6 to 9mm., and in curves of 283m. radius it amounted to 7 to 16 mm.

This enlargement of gauge—due to lateral yielding—was in both cases distributed almost uniformly over the rails lying opposite.*

Because the axis of the vehicle describes a curved path and the vehicle, in addition to its forward longitudinal motion, has a rotation about a vertical axis, the wheels while rolling slide or are displaced in the direction of their axles. There is consequently at each wheel a resistance due to sliding in addition to the resistance due to rolling. These two kinds of resistance to motion operate *independently* of each other, and demand, during the motion of the wheel from one position to another, the doing of an amount of work made up of parts proportional to the paths rolled-over and slid-over, respectively.

In the motion of the wheel on the rail the size and shape of the surface of contact depends on the wheel-load, on the profiles of the touching surfaces and on the compressibility of their material. Usually, the surface of a tire is not cylindrical, the individual points of the areas in contact are not at equal distance from the axis of the wheel, and have, consequently, unequal angular velocities. The result of this is that it is only the points lying in the running-perimeters that really rest on the rail—and therefore roll; all the others slide.

In this manner each point moves in a definite vertical plane perpendicular to the axle; and therefore, the sliding due to the conical shape of the wheel-tread takes place in a direction perpendicular to the axis of the wheel.

The resistance between wheel and rail in the *straight* is made up, consequently, of the (very small) resistance due to this slide, and of that due to rolling.

But when a vehicle moves through a *curve* the wheels are displaced on the rails in the direction of their axles, and there is a sliding of all the points in the surface of contact in this same direction; and if the distances of the contact-surfaces from the axle, viz., the radii of the contact-circles, in both wheel-pairs, do not stand to each other in the same ratio as the radii of the track rails then there is an additional resistance to motion and in the direction of the vehicle's travel. From the combined effect of these two resistances due to sliding at each wheel there arises a measurable increase in the resistance to traction in the draw-bar, or car-coupling, over that in the straight; and it is this excess which is usually termed Curve-Resistance.

In a rigorous determination of curve-resistance the various modifying effects of the atmosphere, swaying of the vehicle, and of journal-friction should be taken into account, because these affect the resistance of vehicles in Curves quite otherwise than they do in Straights. But since the modifications due to these causes are but small, they have been neglected in the determination of curve-resistance on pages 31 to 37.

The permissibility of such a simplification of the theoretic or rigorous determination of curve-resistance need hardly be insisted upon, and the almost perfect coincidence of the numerical results obtained by the Author on the above assumptions with those of the comprehensive experiments carried out in 1877 and 1878 by von Röckl sufficiently justify it.†

The factors chiefly influencing Curve-resistance are (*a*) the curvature of the track, (*b*) the wheel-base, and (*c*) the relative shape of flange-hollow to the rounded-off top corner of the rail-head.

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* "Stabilität des Gefüges der Eisenbahngleise"; p. 236.

† The results of a work by the Author based on the same principle and published, in 1873, in the "Zeitschrift für Bauwesen", "On the Motion of 4-wheeled Railway Vehicles in Curves," exhibit a similar coincidence of results with the experimental determination of v. Röckl.

See the "Organ für die Fortschritte des Eisenbahnwesens": 1881, page 261; also 1885, Plate VIII.

Less influential is (*d*) the conicity, (or the inclination of the surface of the tread to the wheel-axle,) (*e*) the radius of the wheel, and (*f*) the flange-clearance.* The shape of the flange-hollow at the fore-wheel deserves particular attention, because even with the most favourable shape of the flange *the resistance due to the mutual action of the flange and rail at this wheel amounts to some* 80 % *of the total curve-resistance;* and by a bad shape of flange this resistance may be considerably increased. All attempts to diminish curve-resistance by seeking the theoretically most suitable relative forms of rail-head and wheel-tread are, on this account, doomed to failure, unless they concern themselves directly with this fact.†

The simplest means of reducing the amount of resistance at the fore-wheel, and its accompanying wear, is to *lubricate the flange.*

The calculation of the curve-resistance postulates at the outset a knowledge of the forces acting at the circumference of the wheel. Amongst these forces those are of especial importance which act parallel to the axle of the wheel, since they produce severe torsive effects in the rails, and bring about considerable alterations in the loading of the wheels. The extreme effects which these forces may produce are seen in 3-axle locomotives, where the increase or decrease of the load on a single wheel, under the ordinary conditions of practice, may amount to 25 %.

In Locomotives these transverse forces acting parallel to the axles diminish the adhesion available for traction; and this is particularly noticeable in locomotives having a driving fore-axle. Thus, owing to this cause, at most only 80 % of the adhesion of the F. A. of a three-coupled goods engine is available as tractive force. With this decrease of the useful adhesion is further associated a slide of the driving-wheels in the direction of the locomotive's axis. This slide increases as the effort of the engine increases, and the percentage of curve-resistance of the locomotive is proportionately increased. The curve-resistance of a locomotive is, consequently, always greater, proportionately, than that of the vehicles drawn by it; and is greater, the greater the tractive-force developed.

Not only curve-resistance but also the abrasion of rails and treads is enhanced by an increase of tractive-force in the locomotive's draw-bar. On the other hand, the injurious action of the locomotive on the stability of the track decreases with an increase of the tractive-force, and *vice versâ;* so that it is the *light* locomotive and *not* the hauling one which produces in running through a curve the maximum stresses in the track.

When the wheels are firmly braked the horizontal transverse forces acting at the wheels' circumferences almost completely disappear.

For the rest, the following discussion shows that the horizontal forces caused by the vehicle and producing torsion in the rails decrease as the curve-radius increases.

A knowledge of the magnitudes of the transverse forces acting axially on the wheels during the uniform travel of the vehicle, and especially at the fore-wheel, has a particular importance for the Engineer engaged either in the design of track or in its maintenance, because these forces very sensibly influence the wear and life of the permanent-way; so that, for example, many longitudinal-sleeper systems have completely failed in practice simply from their inability to offer sufficient resistance to such torsive effects.

When running through *sharp* curves the direction of the vehicle's rotation in the horizontal plane is uninfluenced by its slight serpentine movement; and accordingly the directions of the forces acting at the circumference of the wheel remain unaltered.

But when the vehicle runs through a *flat* curve, or in the *straight,* then the forces acting at the circumference or tread of the wheels, parallel to the plane of rotation, change in

[*Note the absence of gauge. Note also that when the track-clearance permits, the H. A. stands radial and therefore rolls conically. If all the axles did the same there would be no "Curve-resistance."—TRANS.]

[† This is doubtless an allusion to the writings of Wöhler—see footnote, p. 38—on the relative shapes of wheel-tread and rail-head, in which he advocated (unsuccessfully) the merits of a peculiar shape of wheel tire proposed by himself.—TRANS.]

direction in definite intervals of time in consequence of the sinuous motion in the track; and they displace the axles of the wheels relatively to the vehicle's axis to an extent determined by the amount of the lateral clearance possible of the axle-boxes in the horns. This change of direction of the wheel-axles, and likewise the displaceability laterally of the vehicle's axis relatively to that of the track, exercise *a very important effect on the travel of the vehicle* in straights or in flat curves.

On this account, therefore, the axle-box clearance in the horns and the flange-clearance in the track have been introduced into the following investigations in order to determine their effect on the vehicle's travel.

Regarding the order of treatment in the following investigations, it is only necessary to remark that single free-running wheels and wheel-pairs are first discussed; then the motion of the vehicle as a whole; and again this latter, both when running by itself, and when forming one of a train.

In order to make as clear as possible the somewhat complex phenomena connected with the motion of vehicles in curves we begin with the condition of things obtaining in sharp curves; and then the relations between wheel and rail deduced therefrom are employed to determine the motion in flat curves and straights.

Fig. 3.

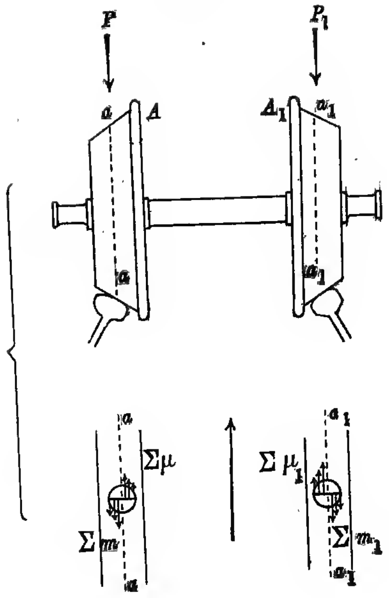
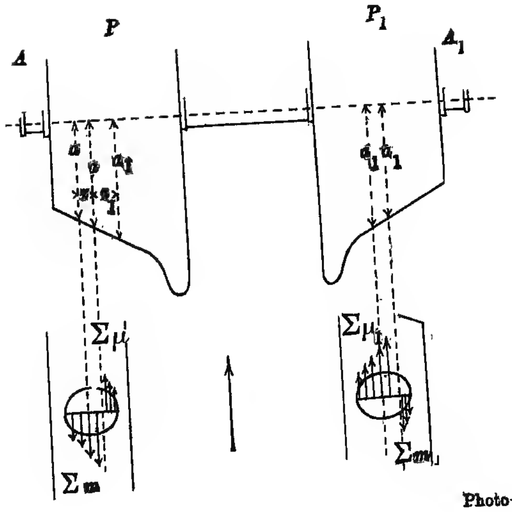


Fig. 4.



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CHAPTER I.

SINGLE WHEEL-PAIR ON THE RAIL.

§ 2.

Position of the Rolling-circle on the treads of the Wheel-tires.

For the determination of the position of the rolling-circle and its radius, and of the amount of the slide-motion occurring at the contact-surface, we have firstly, the general geometrical condition that the rolling-circles of the 2 inner-wheels (rigidly attached to the axle) must lie in a conical surface whose apex coincides with the centre of the instantaneous rotation of the axle: we have also the known shape of the rail and wheel in contact, and the distribution of the axle-load on the wheels.

If the treads of the tires are conical, then all the points in the surface of contact *not* in the rolling-circle, must *slide* on the rail.

Sliding takes place at those points lying in circles which are smaller than the rolling-circle, and in the direction of the vehicle's advance, or forwards: on the other hand, those points which lie in a circle greater than the (conical) rolling-circle slide in the reverse direction, or backwards.

If in **Fig. 3**, $a\,a$ and $a_1\ a_1$ represent the projections on the wheel-treads of the rolling-circles, and if the wheel-pair moves in the direction indicated by the arrow, then the sliding resistances at the wheel left of $a\,a$, and right of $a_1\,a_1$ act concurrently in one direction—in the Fig. they are shown as acting downwards—while those right of $a\,a$ and left of $a_1\ a_1$ act in the contrary direction, and are shown in the Fig. as acting upwards.

Let the statical moments referred to the wheel-pair axis—as shown in the Fig.—of these resistances be represented by $\Sigma\, m$, $\Sigma\, m_1$, $\Sigma\, \mu$, $\Sigma\, \mu_1$; and let M be the sum of the statical moments of the rolling-friction and of the journal-friction acting on the axle; then we must have

$$\Sigma\, m + \Sigma\, m_1 - \Sigma\, \mu - \Sigma\, \mu_1 - M = 0.$$

Let f be the coefficient of sliding friction;

P, the pressure of the wheel A

P_1 „ „ „ A_1

α the mean distance of the surface of contact from the axis of the wheel at the wheel A

α_1 „ „ „ „ „ „ „ „ A_1

Then

$$\Sigma\, m + \Sigma\, \mu = f\, P\, \alpha$$
$$\Sigma\, m_1 + \Sigma\, \mu_1 = f\, P_1\, \alpha_1$$

and consequently the above Equation of Equilibrium becomes

$$2\,\Sigma\, m + 2\,\Sigma\, m_1 = f\, P\, \alpha + f\, P_1\, \alpha_1 + M.$$

Now since M is so small that compared with $f\ P\, \alpha$ and $f\ P_1\, \alpha_1$ it is negligible, we have, sufficiently accurately, from the given conditions,

$$\Sigma\, m + \Sigma\, m_1 = \Sigma\, \mu + \Sigma\, \mu_1$$

and

$$\Sigma\, m + \Sigma\, m_1 = f\, \frac{P\, \alpha + P_1\, \alpha_1}{2}.$$

If the axle moves in a straight line, and if its centre coincides with that of the track, viz, $\alpha = \alpha_1$,

then

$$P = P_1$$

and

$$\Sigma\, m + \Sigma\, m_1 = f\, P\, \alpha = \Sigma\, m + \Sigma\, \mu;$$

whence,

$$\Sigma\, m_1 = \Sigma\, \mu.$$

Now since, owing to the uniformity of conditions at both wheels,

$$\Sigma \mu = \Sigma \mu_1$$
$$\Sigma m = \Sigma m_1$$

therefore

$$\Sigma m = \Sigma \mu$$

and

$$\Sigma m_1 = \Sigma \mu_1.$$

Also, since the arms of the moments Σm and Σm_1, and $\Sigma \mu$ and $\Sigma \mu_1$, for the usual inclination of the treads of $\frac{1}{20}$, differ but slightly from each other, therefore for wheels of equal radius, rigidly fixed on an axle and moving forward in a straight track with equal flange-clearance, the rolling-circle divides the surface of contact of the wheel and rail into two equally loaded halves.

This position of the rolling-circle is maintained even when the load on one of the wheels becomes virtually increased by an impact which does not alter the position of the wheel relative to the rail.

When the axle-middle when running in the straight does not lie directly over that of the track, then α is not equal to α_1; and the rolling-circles a and a_1 do not lie any longer in the middle of the contact-surface.

If, for instance, as in **Fig. 4**, the flange of the wheel A stands further away from the rail than that of A_1, then the pressure on the contact-area is distributed in such a way that $\alpha < \alpha_1$ and, assuming $P = P_1$,

$$f P \alpha < f P \alpha_1.$$

Now in rectilinear motion, $\quad a = a_1$, and thus $\alpha < a < \alpha_1$

and accordingly $\quad \Sigma m > \Sigma \mu$ and $\Sigma \mu_1 > \Sigma m_1$.

Again, because $\quad \Sigma m + \Sigma m_1 = \Sigma \mu + \Sigma \mu_1$

or $\quad \Sigma m - \Sigma \mu = \Sigma \mu_1 - \Sigma m_1$

therefore $\quad \dfrac{\Sigma m - \Sigma \mu}{2} = \dfrac{\Sigma \mu_1 - \Sigma m_1}{2}.$

These last expressions are the moments of the frictional resistances which act on the surfaces of contact between the circles α and a, or α_1 and a_1, respectively.

Consequently, from Fig. 4—

if $$f P \alpha < f P_1 \alpha_1,$$

the rolling-circles a and a_1 both lie to the right of and close to α and α_1 respectively.

Further, from the relation between Σm, Σm_1, $\Sigma \mu$ and $\Sigma \mu_1$ if the shape and the size of the contact-surface of both wheels is the same—which it may be assumed to be, so long as the wheels rest on the conical face of the tread—we see that the distances represented in Fig. 4 by z and z_1 coincide with one another.

Consequently,

$$a = a_1 = \frac{\alpha + \alpha_1}{2}.$$

If the conicity of the tread in contact be $\frac{1}{n}$,

then

$$z = (a - \alpha)\, n = \frac{\alpha_1 - \alpha}{2} n.$$

And since $\frac{\alpha_1 - \alpha}{2} n$ is the displacement of the wheel-pair relatively to the track centre-line, therefore a lies at or in the edge of the contact-area when this displacement amounts to the half-width of the area.

In this position of the wheel-pair relatively to the centre-line of track,

$$\Sigma \mu = 0$$

and

$$\Sigma m = f P \alpha.$$

Thus

$$\Sigma m_1 = f P \frac{\alpha_1 - \alpha}{2} \Sigma m_1$$

and has always a positive value, *i.e.*, the rolling-circle cannot fall outside the contact-area of the wheel for which $P \alpha$ is greatest.

Fig. 5.

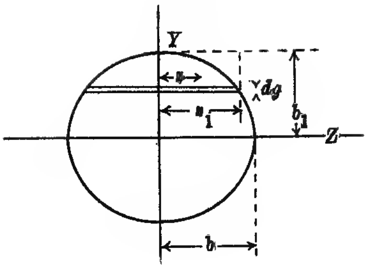


Fig. 6.

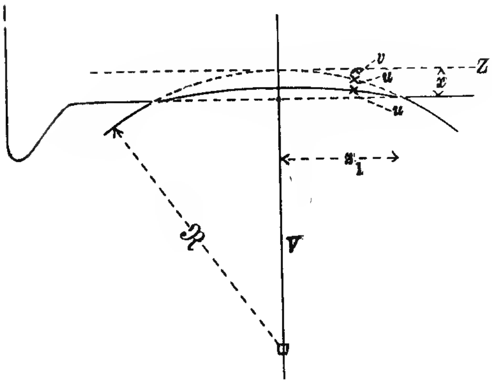
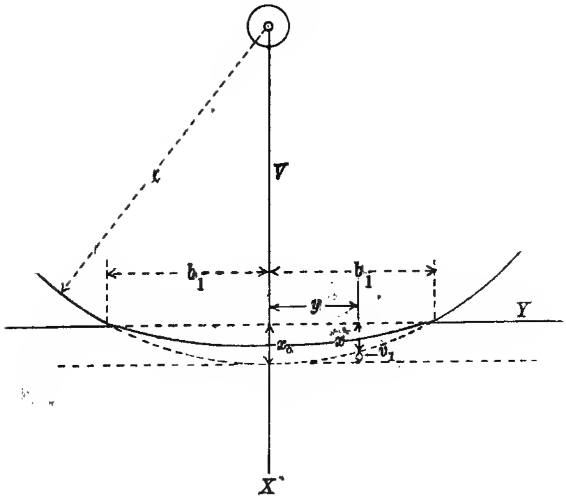


Fig. 7.



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When the displacement of the wheel-pair relatively to the track's centre exceeds the width of the contact-area of wheel A, then the rolling-circle a no longer lies in the contact-area of the wheel.

Now since the width of the contact-area of the wheel is generally less than the flange-clearance, in rectilinear motion of the wheel-pair a complete slide of that wheel can occur at which $P \alpha$ has its minimal value.

The same conditions obtain when the motion of the wheel-pair is not rectilinear but curved; and also if during this motion there is a displacement of the wheel on rail in the direction of its axis.

---

§ 3.
Pressure on the Area of Contact.

Owing to the rounded shape of the rail-head the pressure on the contact-area is distributed in such a way that its intensity increases up to the middle thereof and there attains its maximum.

Let **Fig. 5** be a horizontal plan of the contact-area: let Y be the axis of co-ordinates in the line of the rail, and Z the axis in the line of the axle.

Also,

let $\mathfrak{R}$ be the radius of curvature of the top or table of the rail-head:

$\mathfrak{r}$, the radius of the conical surface of the tread at the heaviest loaded point:

u, the compression at any point in the contact-area of the rail-head:

$a\,u$, the pressure at this point.

Then the load dP on an elemental strip of area of width $2\,z_1$ and of height dy, is

$$d\,P = 2\,d\,y \int_{z=o}^{z=z_1} dzau.$$

If we assume that the elastic compression of the tread is the same as that of the rail-head, and referring to **Fig. 6**—which represents a vertical section perpendicular to the Y-axis through tire and rail at the distance y from the middle of the contact-area, and where x has the value of $2\,u$ for the point $z = o,\ y = y$, in the contact-area—we have

$$u = \frac{x - v}{2},$$

and $$z^2 = 2\,\mathfrak{R}\,v - v^2,$$

or, sufficiently approximately, $$z^2 = 2\,\mathfrak{R}\,v;$$

and consequently, $$d\,P = 2\,d\,y \int_o^x \frac{\mathfrak{R}\,d\,v}{\sqrt{2\,\mathfrak{R}\,v}} \cdot \frac{x - v}{2}\,a,$$

$$= \frac{4a}{3}\sqrt{\frac{\mathfrak{R}}{2}}\,x^{\frac{3}{2}}\,d\,y.$$

Let x_0 be the maximal value of x at the centre of the contact-area; then referring to **Fig. 7**—which represent the vertical section through tire and rail in the direction of the Y-axis—we have the equation

$$P = 2 \int_{x=x_0}^{x=o} \frac{4}{3}\,a \sqrt{\frac{\mathfrak{R}}{2}}\,x^{\frac{3}{2}}\,d\,y$$

in which $x = x_0 - v_1$, and $y^2 = 2\,\mathfrak{r}\,v_1$ are to be inserted; and we thus obtain

$$P = a\,x_0{}^2\,\frac{\pi}{2}\sqrt{\mathfrak{R}\,\mathfrak{r}}. \quad \dots \quad \dots \quad \dots \quad \dots \quad (1)$$

The co-ords, z_1 and y, of any point in the curved boundary of the contact-area, Fig. 5, are connected by the equations $z_1^2 = 2\,\mathfrak{R}\,x$, and $y^2 = 2\,\mathfrak{r}\,(x_0 - x)$.

Consequently, this curve is an Ellipse, of which the equation is

$$\frac{z_1^2}{2\,\mathfrak{R}\,x_0} + \frac{y^2}{2\mathfrak{r}\,x_0} = 1$$

and its area is

$$F = 2\,\pi\,x_0\sqrt{\mathfrak{R}\,\mathfrak{r}}.$$

Whence we obtain for the pressure at its middle-point

$$\mathfrak{p}_0 = a\frac{x_0}{2} = \frac{2\,P}{F}.$$

The assumption on which this deduction is based, *i.e.*, that a is constant, is not exactly true; because the compression of the more-heavily loaded points in the contact-area tends to modify the compression of those less-heavily loaded.

The pressure in the middle of the contact-area is therefore greater than $\frac{2\,P}{F}$, F being the actual size of the contact-area.

Consequently, we have

$$\mathfrak{p}_0 = m\,\frac{2\,P}{F},$$

where m is a coefficient which depends on the material and make of the rail and tire, and which is greater than unity.

Numerical Illustration :—

In an article in the "Organ für die Fortschritte des Eisenbahnwsens:" 1886, page 58; F is shown to be 2·64 *qcm.* when $P = 5500$, *kg*: and in another instance where the wheel-load $P = 3750$ *kg*, $F = 1{\cdot}56$ *qcm*.*

$$\text{Therefore in the first case, } \mathfrak{p}_0 = m.\frac{2.5500}{2{\cdot}64} = m\,4200\,kg,$$

$$\text{and the second, } \mathfrak{p}_0 = m.\frac{2.3750}{1{\cdot}56} = m\,4500\,kg,$$

Therefore in these instances the iron rails were stressed in the contact-area beyond the elastic limit.

If the treads are not worn, then F is a minimum.

For example, on the Lehigh Valley Railroad, with newly-turned-up treads on worn rails, a contact-area of width in the direction of the axle of $\frac{5}{16}$ inch was observed (*Railroad Gazette;* 1882, page 787).

As the length of the contact-area in the direction of the length of the rail was certainly not more than $\frac{5}{16}$ inch, the contact-area was, in this instance much smaller than in the above.

How far the wheel-load affected the question is not determinable from the Paper.

From Eqn. 1 we have

$$\frac{x_0}{2} = \sqrt{\frac{P}{2\,\pi\,a\sqrt{\mathfrak{R}\mathfrak{r}}}},$$

therefore

$$\mathfrak{p}_0 = \frac{ax_0}{2} = \sqrt{\frac{P\,a}{2\,\pi\,\sqrt{\mathfrak{R}\,\mathfrak{r}}}} \qquad \ldots \qquad \ldots \qquad (2)$$

If with a wheel of radius $\mathfrak{r}$, running on a rail of which the top or table-curvature is $\mathfrak{R}$, under a load P, $\mathfrak{p}_0$ is to have the same magnitude as with a wheel of radius $\mathfrak{r}_1$, on a rail of table-curvature of radius $\mathfrak{R}_1$, under a load P_1, then with equal cross-sections of wheel-tire and rail, and the same material, we must have

$$\frac{P}{\sqrt{\mathfrak{R}\,\mathfrak{r}}} = \frac{P_1}{\sqrt{\mathfrak{R}_1\,\mathfrak{r}_1}}$$

$$\text{or} \quad \sqrt{\frac{\mathfrak{R}_1}{\mathfrak{R}}} = \frac{P_1}{P}\sqrt{\frac{\mathfrak{r}}{\mathfrak{r}_1}} \quad \ldots \qquad \ldots \qquad \ldots \qquad (3)$$

$$\text{and if} \quad \mathfrak{R} = \mathfrak{R}_1$$

$$\text{then} \quad 1 = \frac{P_1}{P}\sqrt{\frac{\mathfrak{r}}{\mathfrak{r}_1}} \quad \ldots \qquad \ldots \qquad \ldots \qquad (3a)$$

If, as is assumed in the above, the wheel-tread is conical, so that a plane passing through the axis of the wheel cuts the contact-surface in a straight line, then the magnitude of the radius $\mathfrak{R}$ provides us with a measure of the degree of contact of the unloaded wheel with the surface of the rail in the plane of the cross-section of rail and tire, as represented in Fig. 6.

* $qcm = cm^2$.

But if the original surface of the wheel-tread is hollowed out by wear, then, in addition to the curvature of the rail-head, the curvature of the wheel-tread at the contact-surface has to be considered in its influence on the degree of contact of the two surfaces.

Now it is always possible to imagine the head of the rail as having at the point of contact a curvature of radius $\mathfrak{R}$ such that the curved rail top approaches the (curved) wheel-tread—in the transverse cross-section—as closely as it does in Fig. 6, where the surface of contact of the tire is bounded by a curved line, not by a straight one. Then the Eqns. 1, 2, and 3, hold also for the hollow-worn wheel-tread, if $\mathfrak{R}$ stands for the imaginary value in question.

When a wheel of radius $\mathfrak{r}$ bearing a load P runs on a rail of given cross-section, then by wear a contact-surface is formed of a definite concavity; and the degree of this concavity depends on the curvature of the rail-head, on $\mathfrak{r}$, and on P, and corresponds to a definite magnitude of $\mathfrak{p}_0$ dependent on the material of the wheel. If another wheel of radius $\mathfrak{r}_1$ under a load P_1 runs on a similar rail, and if the wheel be of the same material as the first, then its running-surface acquires a different concavity, but $\mathfrak{p}_0$ has still the same magnitude as before.

The values of $\mathfrak{R}$ and $\mathfrak{R}_1$ corresponding respectively to these two concave running-surfaces stand to one another in the ratio shown in Eqn. 3: and, assuming the running-surfaces of both treads to assume through wear the same cross-section, then the following equation holds;

$$\mathfrak{r}_1 = \mathfrak{r}\left(\frac{P_1}{P}\right)^2.$$

For example; in order that a tread of a wheel loaded with 6580 kg: may not wear more hollow than one of 50 cm radius loaded with 5000 kg. tho radius of the former must be,

$$\mathfrak{r}_1 = 50\left(\frac{6500}{5000}\right)^2 = 86^{cm}.$$

Both these wheels could then run on the same rail without interfering with each other's action in the wearing-down of the rail-head; and $\mathfrak{p}_0$ with newly-turned-up treads would, in both instances, have the same magnitude.*

It may happen that the treads of wheel-tires are not of the shape which would result from their continuous wear under the particular wheel-pressure acting at the moment, and in that case $\mathfrak{p}_0$ is either smaller or greater than if the running-surface of the wheel-treads had the worn shape corresponding to this particular or momentary wheel-pressure.

The first case would occur if a tread worn down under a load P ran for the time being under a load smaller than P.

The other case represents the usual conditions of things in practice when we have fully-loaded wheels and newly-turned-up treads, and especially when running on new rails.

For equal radii and different values of $\mathfrak{p}_0$ we have, from the above expressions,

$$\mathfrak{R}_1 = \mathfrak{R}\left(\frac{\mathfrak{p}_0}{\mathfrak{p}'_0}\right)^2\left(\frac{F_1}{F}\right)^2,$$

where $\mathfrak{R}$, $\mathfrak{p}_0$, and F, and $\mathfrak{R}_1$, $\mathfrak{p}'_0$, and F_1, respectively, represent corresponding connected values:

Numerical Example:—

Put $F = 1{\cdot}562^{qcm}$, $\mathfrak{p}_0 = m\ 4800\ kg$, $F_1 = 2{\cdot}647^{qcm}$, $\mathfrak{p}'_0 = m\ 4200\ kg$, corresponding to the previously cited experimental results; then

$$\mathfrak{R}_1 = 3{\cdot}7\ \mathfrak{R}.$$

Whence we may conclude, that the running or contact-surface of the experimental wheel loaded with 5500 kg, suited or fitted the rail-head in the proportion of 1 : 3·7 better than that of the wheel loaded with 3750 kg.

The ratio of the length to the breadth of the elliptic area of contact is determined from the relation of $\mathfrak{R}$ to $\mathfrak{r}$.

* Agreeably with the above we find that the normal diameter of the vehicle wheels of the Prussian State Railways is 970mm, and that of the driving-wheels of the passenger locomotives, 1730mm. The trailing-wheels of these Engines loaded with about 6400 kg. have a diameter of only 1130mm, and must therefore wear more hollow if $\mathfrak{p}_0$ has the same value as it has for the wheel of a vehicle.

From the preceding it is seen that the greatest width of the above area in the length of the rail is $2\,b_1 = 2\sqrt{2\,\mathfrak{r}\,x_0}$, and crosswise it is $2\,b = 2\sqrt{2\,\mathfrak{R}\,x_0}$: consequently, $b = b_1\sqrt{\frac{\mathfrak{R}}{\mathfrak{r}}}$.

Numerical Example :—

Let $\mathfrak{r} = 45^{cm}$; and for new wheel-treads on new rails, let $\mathfrak{R} = 22{\cdot}5$;
then

$$b = b_1\sqrt{\tfrac{1}{2}} = {\cdot}7 b_1$$

$\mathfrak{R}$ is increased by wear. Put $\mathfrak{R} = \mathfrak{r} = 45^{cm}$, or $\mathfrak{R} = 2\,\mathfrak{r} = 90^{cm}$; then $b = b_1$, and $b = 1{\cdot}4\ b_1$ respectively.

Concurrently with such an increase of the breadth of the area the maximal pressure thereon, by Eqn. 2, would diminish.

When a vehicle runs through a curve, the leading F-wheel does not roll on the conical surface of the wheel-tread, but—as will be shown later on—*in the flange-hollow*. A vertical section, parallel to the axle, through the middle of the area of support gives the curve-line of the flange-hollow, as represented in Fig. 19, of radius r, the lateral rounding of the rail-head top being curved to a circular arc of radius ρ.

In order to make use of Eqn. 2 in calculating $\mathfrak{p}_0$ in this area of contact, we must first ascertain the radius of that circle which in the neighbourhood of the point of contact deviates just as much from its tangent as the circle of radius ρ does from the circular arc of radius r.

This radius is, approximately(1),

$$\mathfrak{R} = \frac{r\,\rho}{r-\rho}.$$

If the radii $\mathfrak{R}$, r, and ρ satisfy the above relation then, when the wheel rests on the tread, the wheel-load F produces, approximately, the same pressure $\mathfrak{p}_0$ as when supported in the flange-hollow.

With new rails $\rho = 14^{mm}$; and for the normally shaped flange-hollow of the Prussian Railways, $r = 15^{mm}$.
Consequently,

$$\mathfrak{R} = \frac{15 \times 14}{1} = 210^{mm}.$$

To obtain r from $\mathfrak{R}$ and ρ, we have from the above equation,

$$r = \rho + \frac{\rho^2}{\mathfrak{R} - \rho} = 14 + \frac{196}{\mathfrak{R} - 14}.$$

Whence we see that r varies but slightly with $\mathfrak{R}$. Giving to $\mathfrak{R}$ the value of the radius for new rails, we obtain values of r only slightly differing from 15^{mm}. Whence, as regards the distribution of wheel-pressure, the rounding of the flange-hollow to a radius $r = 15^{mm}$ corresponds under all circumstances to that of the rail-head of $\rho = 14^{mm}$, when and if the surfaces touch in the particular manner here assumed.

This relation between ρ and r with new rails is, in practice, of theoretic interest only, since it is rapidly altered by a few millemetres' wear.

Examining the increase of load over P, on the flange-hollow, arising from the horizontal curve-pressure during motion in curves, it would be found that the preceding considerations would yield a result only differing from the above in a very insignificant degree.

---

(1) This formula is derived thus: Suppose the Y-axis of a system of rectangular co-ordinates in contact with circles of radii $\mathfrak{R}$, ρ, and r, at the origin: and determine, at the distance y from the x axis, firstly, the deviation of the tangent of the circle $\mathfrak{R}$ from the Y-axis; and secondly, the deviation of the tangents of the circles of radius r and ρ from one another.

Equating these tangent-deviations we obtain,

$$\frac{Y}{\mathfrak{R} - x_1} = \frac{Y}{\rho - x_3} = \frac{Y}{r - x_2}.$$

Substituting for $x_1\ x_2\ x_3$ the values obtained from the appropriate equations of the circles, and neglecting the values of x and y negligible compared with $\mathfrak{R}$ r, and ρ we obtain, approximately,

$$\mathfrak{R} = \frac{r\,\rho}{r-\rho}.$$

§ 4.

The Resistance due to Rolling-Friction.

The energy to be expended in giving a wheel motion for any distance along the rails is proportional to the path described. It is used-up partly in overcoming the sliding-friction in the contact-surface, and partly against the rolling-friction.

The resistance of rolling-friction is a result of the displacement and plastic compression of the material at the contact-surface; and also of the elastic deformation of shape of the wheel-tread and rail; and it may be assumed to be proportional to this latter. Representing the resistance of the rolling-friction by $\mathfrak{w}$, and the energy expended per second in deforming the rail and wheel-tire by $\mathfrak{A}$, the velocity of progression of the wheel by V, and by $\frac{\mathfrak{A}}{m}$ that portion of $\mathfrak{A}$ which is partly consumed in the destruction and permanent change of shape of the material at the common working-surfaces, and which partly disappears in its elastic extension—because the compressed surfaces only completely resume their original shapes when the action of the compressive force ceases, and the energy set free in the squeezing-out is therefore but partially available as a source of impulsion for the motion of the wheel—then

$$\mathfrak{w}\, V = \frac{\mathfrak{A}}{m}; \quad \text{or, } \mathfrak{w} = \frac{1}{m} \cdot \frac{\mathfrak{A}}{V}.$$

For $\frac{\mathfrak{A}}{V}$ — referring to **Fig. 8** — we have the following expression:—

$$\frac{\mathfrak{A}}{V} = 4 \int \int_{o}^{u} dz \; a u \; du = \int 2 a u^2 \; dz.$$

If $u = \frac{x_0 - v}{2}$, and $z^2 = 2 \mathfrak{R} v$, be inserted,

then
$$\frac{\mathfrak{A}}{V} = \frac{8}{15} a \sqrt{\frac{\mathfrak{R}}{2}} x_0^{\frac{5}{2}} \quad \ldots \quad \ldots \quad \ldots \quad (4)$$

and from Eqn. 1

$$\frac{\mathfrak{A}}{V} = \frac{16 P^{\frac{5}{4}}}{15\, 2^{\frac{1}{4}}\, \pi^{\frac{5}{4}} a^{\frac{1}{4}} \mathfrak{R}^{\frac{1}{8}} \mathfrak{r}^{\frac{5}{8}}}$$

Whence it follows that $\frac{\mathfrak{A}}{V}$ for given values of P, $\mathfrak{R}$, and $\mathfrak{r}$, increases with the compressibility of the material of the rail and wheel.

This agrees with the fact that the resistance of rolling-friction diminishes in proportion to the hardness of the material.

Further, from Eqn. 2 it follows that

$$\frac{\mathfrak{A}}{V} = \frac{16\, \mathfrak{p}_0^{\frac{1}{2}}}{15 \pi a^{\frac{1}{2}}} \cdot \frac{P}{\sqrt{\mathfrak{r}}} \quad \ldots \quad \ldots \quad \ldots \quad (5)$$

In this expression the factor $\frac{16\, \mathfrak{p}_0^{\frac{1}{2}}}{15 \pi a^{\frac{1}{2}}}$ depends solely on the material, and on the cross-sections of the rail and wheel-tire.

Representing this factor by k_1 we have

$$\mathfrak{w} = \frac{k_1}{m} \frac{P}{\sqrt{\mathfrak{r}}}$$

And putting
$$\frac{k_1}{m} = k$$

then

$$\mathfrak{w} = k \frac{P}{\sqrt{\mathfrak{r}}} \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (6)$$

For the lever-arm of the rolling-friction we have:—

$$\delta = k\sqrt{r} \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (7)$$

and for its coefficient of resistance :—

$$\mathfrak{w}_1 = \frac{k}{\sqrt{r}} \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (8)$$

This was the form in which the resistance of rolling motion was first expressed by Dupuit, who gave for iron on iron, k = ·0007,[1]

Experiments were made by Poirée and Wood to evaluate the coefficient w_1. Wood allowed single wheel-pairs to run of themselves down a straight descent, and measured the space passed over in known intervals of time. The experimental wheels were ·87^m in diameter, and the maximum velocity attained was 4^m per second. The experiments gave values for w_1 lying between ·001 and ·0016.

Assuming with Wood that the lower value ·001 be the correct one, we obtain for the coefficient :—

$$k = \cdot 001 \sqrt{\cdot 435} = \cdot 00066.$$

But in the above value of w_1 = ·001 is included the sliding-frictional resistance—unavoidably present in the contact-area—together with the resistance arising from the serpentine motion of the wheel-pair.
Consequently, this value of k = ·00066 is too large.

Poirée proceeded in a similar manner to determine the influence of an increase of the diameter of wheel from ·9^m to 1·2^m.

His experiments were carried out (in 1850) on a level straight. He employed a dynamometer and a velocity of 3^m with a wheel-load of 3 tonnes; and found w_1 = ·00091, when 2 r = ·9^m ; and w_1 = ·00075, when 2 r = 1·2^m. Whence—neglecting the fact that these coefficients are influenced by the sliding-friction—we have

$$k = \cdot 0009 \sqrt{\cdot 45} = \cdot 00060,$$

and

$$k = \cdot 00075 \sqrt{\cdot 60} = \cdot 00058.$$

The discrepancy of these values of k is to be attributed mainly to the fact that the resistance of sliding-friction in the contact-area, as also that due to the deviation of the vehicle's path from the straight, diminishes as r increases; thus for r = ·45^m. it is greater than when r = ·6^m. The value of k above found yields the following values of δ :

Wood.	Poirée.	
r = ·435^m.	r = ·45^m.	r = ·60^m.
δ = ·000435^m.	δ = ·000405^m.	δ = ·00045^m.

According to Grashof,[2] for railway vehicle wheels of about ·5^m radius, δ varies from ·0005^m to ·00055^m.

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(1) Couche :— "Voie, matériel roulant, et exploitation technique des chemins de fer :" Vol. 3, pp. 602—617. According to Dupuit, k for metalled roads is ·03 : for wood on wood, ·0011 : and for iron on iron, ·0007. The (earlier) expression of Coulomb for this coefficient is of the form $w_1 = \frac{k}{r}$.

(2) Grashof—"Theoretische Maschinenlehre :" Vol. 2, page 297. The lever arm δ given by Coulomb, Rondelet, Poncelet, Pambour, Rittinger, Weisbach, and others, is

1. For cast-iron rollers of about ·5^m diam. on iron rails, δ = ·48^{mm}.
2. Railway vehicle wheels of about 1^m diam. on cast-iron rails, δ = ·5^m to ·55^{mm}.
3. For cast-iron rollers on a granite road, δ = 1^{mm}.
4. For wooden rollers on chisel-dressed stone, δ = 1·3^{mm}.
5. For wooden rollers on wood, δ = ·5 to 1·5^{mm}.

Here Grashof's remarks on the nature of rolling-friction are worthy of notice. He says, § 81 ;—"Up to the "present it has been usually considered satisfactory to explain rolling-friction as a consequence partly of the roughness "of the surface, and partly of the deformation of the roller and of its track brought about by the action of the vertical "pressure p. Thus the rolling motion is a succession of overturnings, that is, of rotations about axes, which in "the direction of their sense or sequence (of overturnings) almost coincides with the momentary line of direction of p. "Meanwhile an exhaustive experimental investigation into the nature of this resistance has been carried out by "Professor Osborn Reynolds in 1875.

"According to this investigation rolling-friction consists mainly of relative sliding movements ; and conse-"quently this resistance is to be regarded as *identical in kind* with friction, strictly so called."

This view which contradicts the fundamental postulates on which our investigations of w_1 is based, is not supported by the conclusion deduced by Grashof and others from these experiments of Reynolds.

On the contrary, these experiments serve, with the assistance of quite unexceptionable data, to support the view of the nature of rolling-friction which Reynolds is supposed to have discredited.

Fig. 8.

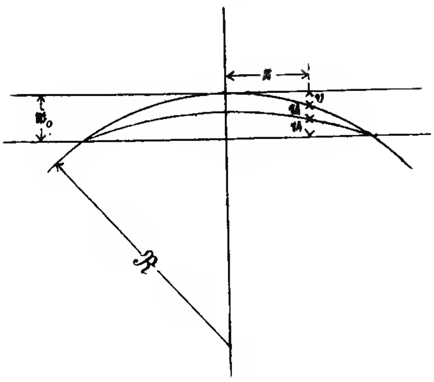
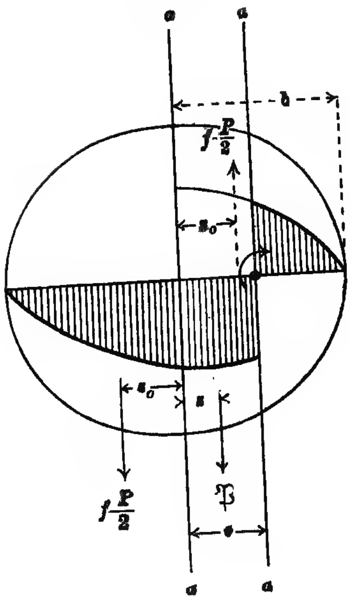


Fig. 9.



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§ 5.

The Resistance arising from the Slide at the Surface of Contact during Rolling.

It has already been stated that the conical shape of wheel-treads causes a slide or skid at all points of the contact-surface not lying in the running-perimeter, wherever this latter may be at the time.

If this rolling-circle happens to lie in the middle of the contact-surface and if z_0 is the distance of the resultant of all the frictional resistances lying on one side of the rolling-circle, then the moment of these resistance is

$$M = f P z_0.$$

If, on the other hand, the rolling-circle lies, as shown in **Fig. 9**, in the position $\alpha\,\alpha$ at a distance c from the centre of the area, then the moment of resistance is

$$M_1 = M + 2f \mathfrak{P}\,(c - z),$$

where $\mathfrak{P}$ is the sum of the resistance lying between $a\,a$ and $\alpha\,\alpha$, and z is the distance of their resultant from $\mathfrak{a}\,\mathfrak{a}$.

The expression for the length of the lever arm z_0 is given—on the assumption that the coefficient a is constant for all points in the area—by the following expression:

$$z_0 = \frac{16}{15\pi}\sqrt{2\,\mathfrak{R}\,x_0} \quad = \text{about}\,\frac{b}{3}:$$

and therefore,

$$M = f P \frac{b}{3}.$$

Consequently, to overcome this moment a force acting in the axis of the wheel is requisite, namely,

$$\mathfrak{w} = \frac{M}{n\,\mathfrak{r}} = \frac{f\,b}{3\,n\,\mathfrak{r}}\,P,$$

where $\frac{1}{n}$ is the tan. of the angle which the contact-surface makes with the axle, *i.e.*, the conicity.

The resulting Coefficient of resistance is, therefore,

$$\mathfrak{w}_2 = \frac{f\,b}{3\,n\,\mathfrak{r}}.$$

Numerical Example:

If $b = \cdot 6^{cm}$, $f = \cdot 25$, $n = 20$, and $\mathfrak{r} = 45^{cm}$,

then $\qquad \mathfrak{w}_2 = \frac{1}{18000}.$

In the above expression for M_1 the second term may be regarded as a part of M dependent on the position of the rolling-circle relatively to the middle of the contact-area; consequently it can be put into the form

$$M_1 = M\,(1 + \epsilon)\,;$$

where ϵ is a coefficient depending on z. Finally, we have the more general expression for the coefficient of friction $\mathfrak{w}_2$ corresponding to M_1 as follows:

$$\mathfrak{w}_2 = \frac{1+\epsilon}{n\,\mathfrak{r}\,P}\,M = \frac{1+\epsilon}{3\,n\,\mathfrak{r}}\,f\,b$$

or,

$$\mathfrak{w}_2 = \frac{D}{\mathfrak{r}} \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (9)$$

§ 6.

Resistance of Wheels in the Straight.

The friction occurring during the motion of a vehicle is expressed, consequently, by the coefficient,

$$\mathfrak{w} = \mathfrak{w}_1 + \mathfrak{w}_2$$
$$= \frac{k}{\sqrt{x}} + \frac{D}{x};$$

in which expression k and D are independent of x.

When vehicles are in motion, in addition to the resistance occurring between the wheel and rail, there is further to be taken into account the work done in causing the rails and sleepers to deflect and oscillate, and in the elastic compression of the ballast: and there is also the resistance arising from the swinging or meandering motion of the axles.

CHAPTER II.

THE MOTION OF 4-WHEELED VEHICLES IN CURVES.

§ 7.

The Position of a Vehicle in a Curve.

It has already been pointed out that the motion of a vehicle in a curve is always associated with a rotation about a vertical axis, and that this rotation increases uniformly with the curvature of the path. The force operating this rotation is only in a very small degree due to the tension of the couplings; it is mainly brought about by the mutual action of the wheels and rails on each other: and the magnitude and direction of this action are the more definite and pronounced the severer the curvature of the track.

For this reason it appears desirable in the following discussion of the laws governing the action of the forces between wheel and rail—which forces depend on the type of vehicle and on the character of the track—to take as the starting point of our investigation the condition of things prevailing in *sharp* curves.

It is of course understood that in doing this we shall neglect the unavoidable disturbances of uniform motion arising from defects in track and in the vehicle, and from any other chance sources of irregularity.

When a vehicle moves on the rails in the direction of its longitudinal axis with a velocity V, in a curve of radius R, it rotates at the same time with a velocity $\frac{V}{R}$ about an axis perpendicular to the direction of V and inclined to the vertical at an angle equal to the superelevation of the outer-rail. The position of the rotation-axis of the vehicle is determined by the following considerations—which apply to a 4-wheeled vehicle having parallel axles. Omitting for the present from consideration the influence of the forces acting through the couplings at both ends of the vehicle when running in a train, and assuming that the superelevation of the outer-rail is that exactly proper to the velocity V of the C. G. of the vehicle and, consequently, that there is no surplus force to bring about a lateral displacement of the vehicle neither in the centripetal nor in the centrifugal direction; then the position of the vehicle depends solely on the action of the forces acting between the wheels and rails.

Imagine the resultants of the forces acting on each individual wheel to be projected, on the horizontal plane perpendicular to the rotation-axis of the vehicle, and these forces again resolved in the direction of the velocity and at right-angles thereto. We obtain thus 8 forces which, under the above suppositions, are in equilibrium. Let it now be assumed, provisionally, that the 4 forces lying in the direction of V are in equilibrium. Then we have to deal solely with the components lying in the direction of the (projected) wheel-axles.

Fig. 10.

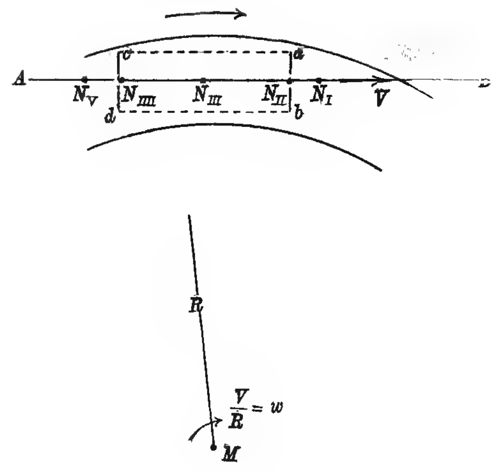


Fig. 11.

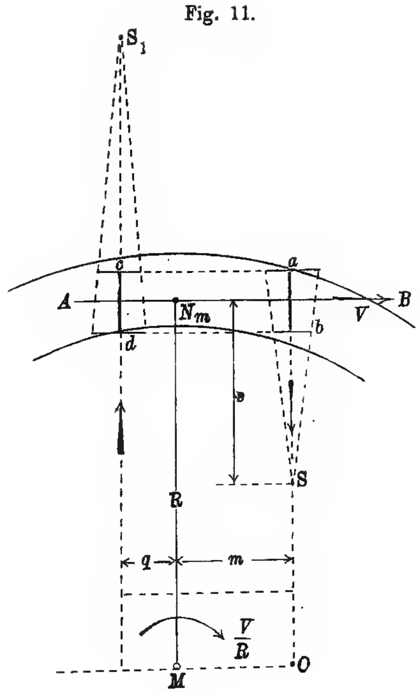
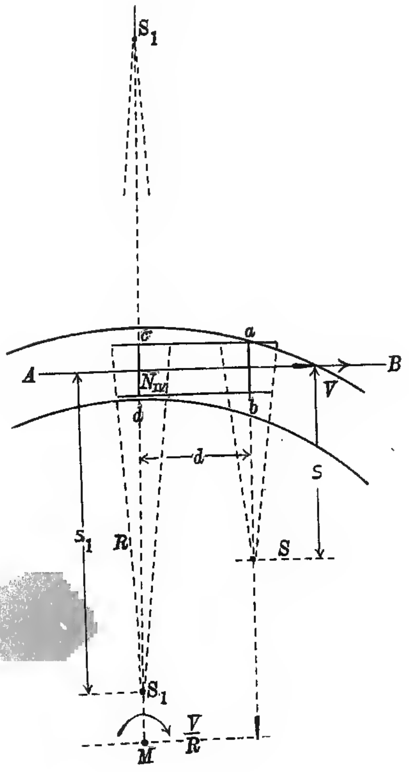


Fig. 12.



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In **Fig. 10** let $a\,b$ be the projection of the F. A., $c\,d$ that of the H. A., and $A\,B$ that of the vehicle's longitudinal axis. Then we have to determine whether the rotation-axis of the vehicle lies in front of, or in the F. A., or between the two axles, or in the H. A., or behind it. These positions are indicated in the Fig. by N_1, N_2, N_3, N_4, N_5.

If we now examine what the displacements of the 4 wheels on the rails must be if the rotation-axis lie in N_1, N_2, or N_5, then it is clear that the rotation-axis cannot lie in any one of these points if the vehicle's rotation is to be brought about solely through the action of the forces acting in the direction of the axles $a\,b$ and $c\,d$—which is our starting assumption. Consequently, the above are inadmissible as rotation-axes, and the points N_3 and N_4 are the only possible ones.

If N_3 be the rotation-axis of the vehicle lying between the F. A. and H. A., then the wheels are compelled to slide in the direction indicated by the arrows—**Fig. 11**—which on the initial assumptions is only possible if the F. A., $a\,b$, runs against the outer-rail and the H. A., $c\,d$, against the inner.

When N_4 is the rotation-axis, viz., in the H. A., then only the F. A. moves, and that in its own direction, as represented in **Fig. 12**.

The points N_3, N_4 are the feet of the perpendiculars dropped from the curve-centre M on to the longitudinal axis of the vehicle. The rectangle $a\ b\ c\ d$ rotates about these points in both positions, with the angular velocity $\frac{V}{R}$, and at the same time the wheel-pairs slide in the direction indicated by arrows in the Figs. 11, 12.

Now suppose—Fig. 11—the inner-rail to be gradually withdrawn from the outer-rail—in other words, the gauge to be widened; then the vehicle-axis assumes a new position relatively to the axis of the track, the point N_3 moving closer to the H. A. until this axle again runs in contact with the inner-rail. If this enlargement of the gauge be carried still further until N_3 reaches the H. A.—the position Fig. 11 thus becoming that of Fig. 12—then the flange of the inner H.-wheel becomes tangential to the inner-rail, and the H. A. stands radially in the curve; under these circumstances it is then solely the F. A. that is deviated axially by the pressure of the outer-rail. *A continuation of the enlargement of gauge produces no further change in position of the H. A.*

Whether the H. A. stands radial, or whether the rotation-axis lies in front of the H. A. depends—on the above assumptions—only on the magnitude of the flange-clearance.

Let σ represent this clearance, and d the wheel-base of the vehicle; then

when $\sigma \overset{=}{>} \frac{d^2}{2R}$, the H. A. stands radially;

and when $\sigma < \frac{d^2}{2R}$, the rotation-axis lies in front of it.

These conclusions regarding the position of a vehicle in a curve are based on the assumption that the resolved forces acting between wheel and rail *parallel* to the longitudinal axis of the vehicle, and the *tractive force* in the couplings, have no influence on the position of the vehicle. Actually, this condition of things seldom or never occurs. In most instances these forces—here omitted from consideration—are the very ones which *do* decide the actual position of the vehicle; and this is particularly the case in *flat* curves. In such curves from the action of *these forces* and without any assistance from the pressure due to the running-in-contact of the inner H.-wheel with the inner-rail, it is possible for the vehicle to rotate about an axis lying between the F. and H. Axles *even though* σ *be* $> \frac{d^2}{R^2}$.

It is also quite possible under the action of the above forces that the rotation-axis may lie behind the H. A., if the vehicle runs through a *sharp* curve in which $\sigma > \frac{d^2}{2R}$.

But in the following investigations these exceptions to the rule above given may be neglected without noteworthy influence on the correctness of the results.

§ 8.

The Sliding of the Wheels on the Rail.

Owing to the running of the outer F-wheel in contact with the outer-rail and to the accompanying displacement of the axle's middle relatively to the track-centre, the apex S of the rolling-cone of the F. A. falls in both Figs. 11 and 12 on the concave side of the curve; while for similar reasons the apex S_1 of the rolling-cone of the H. A., when in the position of Fig. 11 always falls on the convex side. But for the position of the vehicle represented in Fig. 12 the position of S_1 depends on the amount of the flange-clearance σ; and it may lie either on the concave side or on the convex.

The distance of the rolling-cone apex from the axis of the vehicle varies with the change in position of the axles relatively to the track centre.

The motion of the vehicle's F. A. about the centre of the curve can be resolved into a rectilinear motion of translation in the line of the axle, and another of rotation about the point O as centre, —see Fig. 11, where O is the point of intersection of a straight line through the curve's centre M, parallel to the vehicle's axis, with the prolonged direction of the F. A.

The velocities of these motions are $d\varpi$ and ϖ, respectively, where ϖ is the angular-velocity of the vehicle about the curve-centre.

Were the F. A. completely free it would not rotate with the velocity $\varpi = \frac{V}{R}$, nor would it rotate about the point O; its rotation would be conical, with the velocity $\frac{V}{\rho}$ about the cone apex S, and it would thus be greater than the actual rotation by the velocity $\Delta = \frac{V}{\rho} - \frac{V}{R}$. But since the axle, by its connection with the vehicle, is compelled to rotate with the velocity ϖ, the F. A. wheels must slide longitudinally on the rails with a resultant velocity,

$$s\Delta = Vs\left(\frac{1}{\rho} - \frac{1}{R}\right) \quad \dots \quad \dots \quad \dots \quad (10)$$

in the direction of the longitudinal axis, if ρ be the height of the rolling-cone, and s the distance apart of the centres of the F-wheels' contact-areas, *i.e.*, of the rolling-perimeters.

Omitting now from consideration the influence of the contact-surfaces on the position of the slide-paths thereon, and assuming the whole wheel-load concentrated at the centre of the contact-surfaces, then the slide-path of the individual wheels is obtained as follows.

Because both wheels experience the same displacement $d\varpi$ in the axle-direction they likewise participate equally in the velocity $s\Delta$, *i.e.* if wheel-load and wheel-radius are the same for both wheels—what is here presupposed. Each wheel slides, therefore, on its rail with the resultant velocity

$$\mathfrak{w} = \sqrt{(d\varpi)^2 + \left(\frac{s\Delta}{2}\right)^2}. \quad \dots \quad \dots \quad \dots \quad (11)$$

The radius ρ is given by the following, — see **Fig. 13**, in which r and r_1 are the radii of the rolling (cone) circles :—

$$r - r_1 : s = \frac{r + r_1}{2} : \rho,$$

whence

$$\rho = \frac{\frac{r + r_1}{2}\, s}{r - r_1} :$$

and if we put x for

$$\frac{r + r_1}{2}$$

then

$$\rho = \frac{x\, s}{r - r_1} \quad \dots \quad \dots \quad \dots \quad \dots \quad (12)$$

Fig. 13.

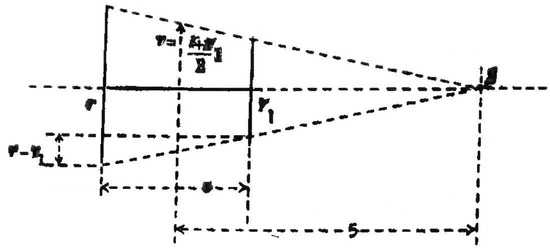


Fig. 14.

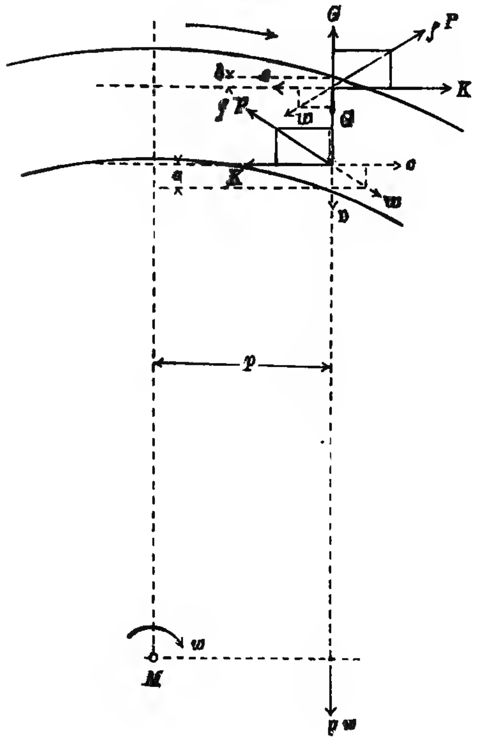
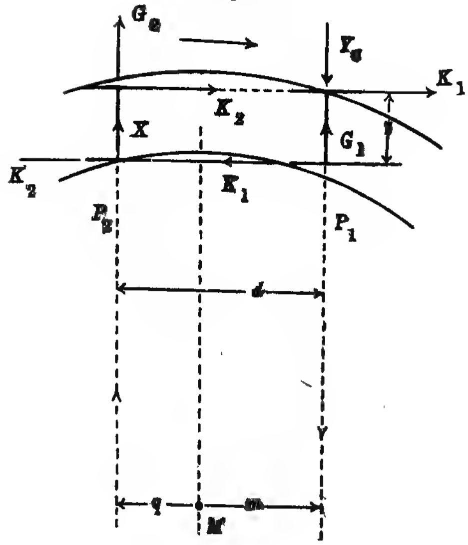


Fig. 15.



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If when the H. A. is radial, and $\sigma =$ the *total* clearance, $b =$ the amount of clearance between the outer-wheel flange and the outer-rail, and $a = (\sigma - b)$ that between the inner-wheel flange and the inner-rail; then if r and r_1 be the radii of the inner, and outer-wheels, respectively, and $\frac{1}{n}$ the conicity, their difference is

$$(r - r_1) = \frac{a - b}{n},$$

or

$$= \frac{\sigma - 2b}{n},$$

whence,

$$p_1 = \frac{n\,x\,s}{\sigma - 2b} \quad \ldots \quad \ldots \quad \ldots \quad (12a)$$

---

§ 9.

The Horizontal Pressure of the Fore-wheel against the Rail.

If a wheel-pair in steady motion occupies the position outlined in **Fig. 14**; and if

$c =$ velocity of the slide of the wheels in the line of the vehicle's motion *i.e.*, longitudinally:

$v = p\,\omega$, the velocity with which they slide axially, *i.e.*, across the rails:

$P =$ the wheel-load: $f =$ coefficient of sliding friction:

$K =$ the component of the sliding frictional-resistance fP corresponding to the slide velocity c:

$G =$ the component of this resistance corresponding to the velocity v:

then we have from Eqns. 10 and $12a$:—

$$c = \frac{V}{2}\left(\frac{\sigma - 2b}{n\,x} - \frac{s}{R}\right) \quad \ldots \quad \ldots \quad \ldots \quad (12b)$$

and from Fig. 14,

$$K = fP\,\frac{c}{\sqrt{c^2 + v^2}}$$

and

$$G = fP\,\frac{v}{\sqrt{c^2 + v^2}}.$$

Substituting in these expressions the above values of c and v, we obtain

$$K = fP\,\frac{\frac{1}{2}\left(\frac{R\sigma - 2bR}{n\,x} - s\right)}{\sqrt{p^2 + \frac{1}{4}\left(\frac{R\sigma - 2bR}{n\,x} - s\right)^2}} \quad \ldots \quad \ldots \quad (13)$$

$$G = fP\,\frac{p}{\sqrt{p^2 + \frac{1}{4}\left(\frac{R\sigma - 2bR}{n\,x} - s\right)^2}} \quad \ldots \quad \ldots \quad (14)$$

These equations will also give the values of K and G for the particular position of the vehicle shown in Fig. 11, if we put for the F. A. $p = m$, $b = 0$; and for the H. A. $p = -q$, $b = \sigma$. To adapt these equations to the position of vehicle represented in Fig. 12,—H. A. radial—we must put $p = d$, and $b = 0$, for the F. A; and $p = 0$ and $b = \frac{d^2}{2R}$, for the H. A.

When the vehicle runs through a curve with the outer F-wheel and the inner H-wheel against their respective rails it is at *these wheels alone* that the forces originate which produce the vehicle's rotation about a vertical axis. These forces act in the line of the axles, and in such wise that the outer F-wheel, and the inner H-wheel continue to mount, by means of the flange-hollow, on to the round corner of the rail-head until the contact-surface reaches such an inclination that the wheel-load overcomes the resistance supporting the wheel in this position on the rail, with the result that a continuous slipping or sliding downwards of the wheels on the rails takes place.

The magnitude of these forces acting at the wheels and effecting the rotation of the vehicle, is obtained from Eqns. 13 and 14 thus:—

In **Fig. 15**, Y_a is the effective pressure of the outer F-wheel;

consequently— $$Y_a\, d + K_1\, s + K_2\, s - G_1\, d = 0:$$

therefore $$Y_a = G_1 - K_1 \frac{s}{d} - K_2 \frac{s}{d}$$

The value of K_2 is given by Eqn. 13, by putting $b = \sigma$, and $p = -q$. Consequently,

$$Y_a = f P_1 \underbrace{\frac{m - \frac{s}{2d}\left(\frac{R\sigma}{n_1 x_1} - s\right)}{\sqrt{m^2 + \frac{1}{4}\left(\frac{R\sigma}{n_1 x_1} - s\right)^2}}}_{\text{Fore Axle.}} + f P_2 \underbrace{\frac{\frac{s}{2d}\left(\frac{R\sigma}{n_2 x_2} + s\right)}{\sqrt{q^2 + \frac{1}{4}\left(\frac{R\sigma}{n_2 x_2} + s\right)^2}}}_{\text{Hind Axle.}} \quad \ldots \quad \ldots \quad (15)$$

In the above, P_1, n_1, x_1, refer to the F. A.

P_2, n_2, x_2, „ „ H. A.

When the H. A. stands *radially*, so that $q = o$, and $m = d$, then instead of the above Eqn. 15 we have

$$Y_b = f P_1 \underbrace{\frac{d - \frac{s}{2d}\left(\frac{R\sigma}{n_1 x_1} - s\right)}{\sqrt{d^2 + \frac{1}{4}\left(\frac{R\sigma}{n_1 x_1} - s\right)^2}}}_{\text{Fore Axle.}} \pm \underbrace{f P_2 \frac{s}{d}}_{\text{Hind Axle.}} \quad \ldots \quad \ldots \quad (16)$$

The *minus* sign in the last term holds for the case in which—as in Fig. 12—the apex S_1 lies between the inner-rail and the centre of curve.

The value of q to be substituted in Eqn. 15 is given by the relation—obtained from Fig. 15—

$$\frac{(d-q)^2}{2R} - \frac{q^2}{2R} = \sigma$$

$$\therefore \quad q = \frac{d}{2} - \frac{R\sigma}{d}. \quad \ldots \quad \ldots \quad \ldots \quad (17)$$

The minimum value of $R\sigma$, *i.e.*, when $q = o$, (viz., H. A. radial) is therefore

$$R\sigma = \frac{d^2}{2}. \quad \ldots \quad \ldots \quad \ldots \quad (17a)$$

For Passenger vehicles, and Goods waggons and Secondary-Line locomotives of the Prussian State Railways we obtain by means of the above the following Table:

Vehicle.	d.	for $q = o$, $R\sigma$
Passenger cars ...	5^m	12·5
Goods waggons ...	4^m	8·0
Sec.-Line locos ...	$2{\cdot}5^m$	3·13

If $R\sigma$ continuously increases beyond its value for which $q = o$, then the H. A. rolling-cone apex lying on the convex side of the curve removes itself to infinity, and afterwards appears on the concave side of the curve: and for a certain value of $R\sigma$ it coincides with the curve-centre M.

This value is given by the condition that $\rho_2 = R$.

Now by Eqn. 12a

$$\rho_2 = \frac{n_2 x_2 s}{\sigma - 2b} = \frac{n_2 x_2 s}{\sigma - \frac{d^2}{R}}:$$

and consequently, when $$\rho_2 = R$$

then $$R\sigma = d^2 + n_2 x_2 s \quad \ldots \quad \ldots \quad \ldots \quad (18)$$

and for this value of $R\sigma$, $$C_2 \text{ and } K_2 = 0.$$

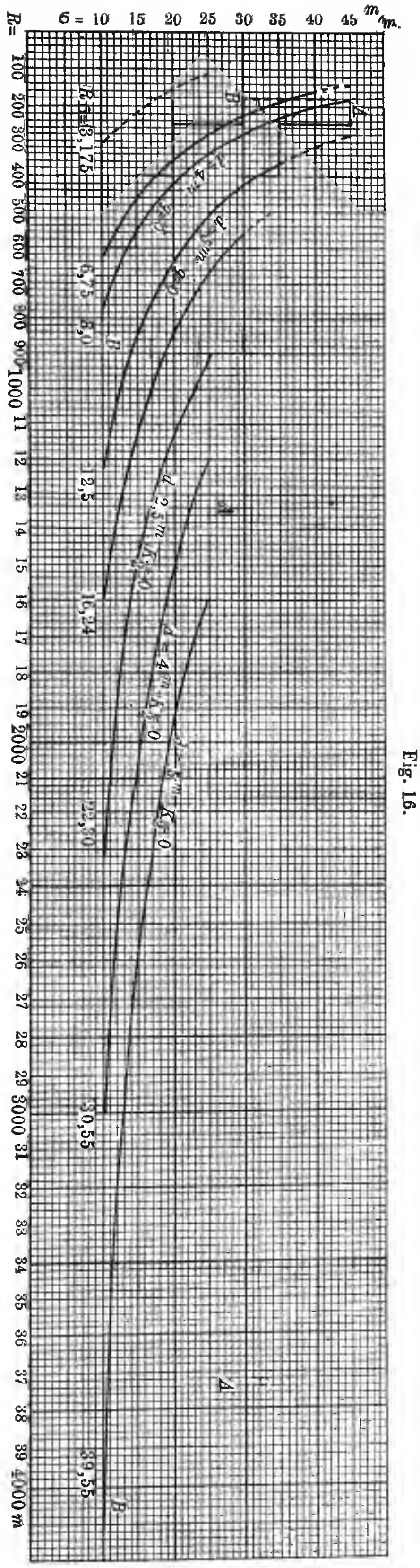


Fig. 16.

Fig. 17.

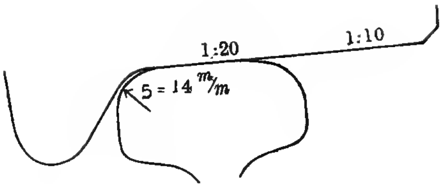
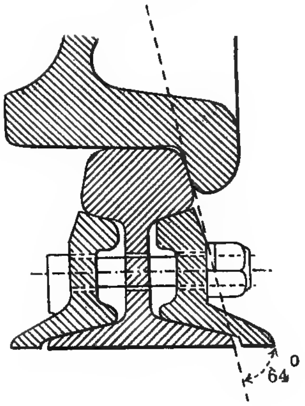


Fig. 17a.



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The following Table exhibits the value of $R\sigma$ given by Eqn. 18 for different vehicles and data given therein.

Vehicle.	$d.$	n_2	r_2	s	$\rho_2 = R$ for $R\sigma$,
Passenger cars ...	5^m	20	$\cdot 485^m$	$1{\cdot}5^m$	39·55
Goods waggons ...	4^m	20	$\cdot 485^m$	$1{\cdot}5^m$	30·55
Sec.-Line locos. ...	$2{\cdot}5^m$	20	$\cdot 535^m$	$1{\cdot}5^m$	22·3

Thus we see that *the magnitude of $R\sigma$ determines the position of the H. A.* and of the position of the apex of its rolling-cone; and consequently also the value of K_2; and this magnitude must therefore be carefully considered when calculating definite values of Y from the Eqns. 15 and 16.

Thus when $R\sigma < \frac{d^2}{2}$ then $q > o$, $v_2 < o$, $c_2 < o$ and $K_2 < o$ $\}$ and Y is to be found from Eqn. 15.

On the other hand, if

$R\sigma \geqq \frac{d^2}{2}$,

then $q = o$

$v_2 = o$

and c is negative, so long as ρ_2 is negative

or c is positive and $> R$

Consequently, if $\frac{d^2}{2} \leqq R\sigma < (d^2 + n_2 r_2 s)$, $\}$ Y is to be found from Eqn. 16, and the second term is to be taken *positive*.

If it happens that $R\sigma = \frac{d^2}{R} + n_2, r_2 s$

then $\rho_2 = R,$

and $K_2 = O$ $\}$ and then either Eqn. 15 or 16 will give Y, neglecting the second term.

Finally, if $R\sigma > d^2 + n_2 r_2 s$

then $v_2 = o$, $c_2 > o$, $K = + f P_2$ $\}$ and Y is to be found from Eqn. 16 with the *minus* sign before the second term.

If we represent the curve-radii R as abscissœ and above them plot the corresponding maxima and minima values of the flange-clearance, σ, as ordinates, then we obtain the two line *A A*, and *B B*, represented in **Fig. 16**, which are 15^{mm} apart. Again, similarly plotting off at these radii, the values of σ which, for the wheel-bases given in the two preceding Tables, make q and K_2 each zero, [for example: — when $d = 5^m$, those given by the Eqns., $\sigma = \frac{d^2}{2R} = \frac{12{\cdot}5}{R}$, and $\sigma = \frac{d^2 + n_2 r_2 's}{R} = \frac{39{\cdot}55}{R}$] we shall obtain the curved lines in the Fig. lying between *AA* and *BB*.

These curved lines divide the area bounded by *A A* and *B B* in such a way that every point on the *left* of the line for $q = 0$ corresponds to one for $q > o$, and for $K_2 = o$ corresponds one to $K_2 < o$; similarly, on the *right* of the line every point for $q = o$ corresponds to one for $q = o$, and for $K_2 = o$ to one for $K_2 > o$.

If $n_1 r_1$ and $n_2 r_2$ are independent of R, then from Eqns. 15 and 16, Y varies solely with $R\sigma$: and so the curve-lines in Fig. 16, give at the same time the geometrical point at which definite constant values of Y occur. When the wheel-treads are of the usual shape as represented in **Fig. 17,** then $n_1 r_1$ and $n_2 r_2$ are not independent of R; because such treads, owing to their shape, are able, under circumstances, to seek out another point of support on the rounded top-corner of the rail-head. But with worn treads these quantities are independent of R, and especially with treads of such shape as renders impossible the wheel's seeking out a support-area on the top-corner of the rail-head corresponding to the value of Y: as for instance, with such shapes of rail and wheel as are figured in **Fig. 17***a*, which have been recently adopted on the Lehigh Valley Railroad. * With this profile of rail and tire, the rails stand vertical, the middle of the tread is cylindrical, and a lateral contact of the flange with the rail occurs at an angle of 64° with the horizontal.

* "The Railroad Gazette": 1882. p. 787.

§ 10.

The Inclination to the Axle of the Contact-Surface of the Fore-wheel.

The inclination of the contact-surface on which the outer F-wheel slides downwards depends on the pressure Y, due to the load on this wheel, to be overcome, and on the direction in which the contact-surface slides on the rail.

Let A be the point in this surface at which P_1, Y, and K_1 may be conceived as acting. If the plane in which the contact-surface lies be intersected by two vertical planes intersecting in A, of which one is parallel to the axle of the Fore-wheel and the other contains the direction of the resultant slide motion of the point A (in the circumference of the wheel) then we obtain the line $A\ B$, in the first-mentioned vertical plane, and the line $A\ D$, of **Fig. 18** in the second. Further, if the points B, C, D, lie in a horizontal plane then the angle α gives sufficiently correctly the inclination to the axle of the plane $A\ B\ D$ in which the sliding takes place.

The direction of P coincides with the line of intersection, $A\ C$, of the vertical planes, the sliding takes place in $A\ D$, and Y lies in the plane $A\ B\ C$. By appropriate resolution of P and Y, the forces acting in the plane $A\ B\ D$ are obtained, as represented in Fig. 18. Consequently,

$$Y \cos\alpha + f\,(P_1 \cos\alpha + Y \sin\alpha) \cos\beta - P_1 \sin\alpha = 0$$

and $\therefore$

$$\tan\alpha = \frac{Y + f\cos\beta\, P_1}{P_1 - f\cos\beta\, Y} \qquad \ldots \qquad \ldots \qquad (19)$$

The angle β included by the lines $A\ B$, $A\ D$, is always less than the angle γ between $C\ B$ and $C\ D$: however, the exactitude of the following investigation will not be impaired if we put β for γ.

The angle β is given by the equation,

$$\cos\beta = \cos\gamma = \frac{v}{w} = \frac{m}{\sqrt{m^2 + \frac{1}{4}\left(\frac{R\,\sigma}{n_1 x_1} - s\right)^2}} \qquad \ldots \qquad (20)$$

or from Eqn. 17

$$\cos\beta = \frac{\frac{d}{2} + \frac{R\,\sigma}{d}}{\sqrt{\left(\frac{d}{2} + \frac{R\,\sigma}{d}\right)^2 + \frac{1}{4}\left(\frac{R\,\sigma}{n_1 x_1} - s\right)^2}} \qquad \ldots \qquad (20a)$$

To determine n_1 we have—from **Fig. 19**—

$$\frac{\sigma}{n_1} = \frac{\sigma}{n} + z: \quad \text{and } z = r\,(1 - \cos)\,\alpha$$

and so

$$n_1 = \frac{n\sigma}{\sigma + nr\,(1 - \cos\alpha)} \qquad \ldots \qquad \ldots \qquad \ldots \qquad (20b)$$

consequently,

$$\frac{R\sigma}{n_1\, x_1} = \frac{R\,\sigma}{n\, x_1} + \frac{Rr}{x_1}(1 - \cos\alpha). \qquad \ldots \qquad \ldots \qquad \ldots \qquad (21)$$

Fig. 18.

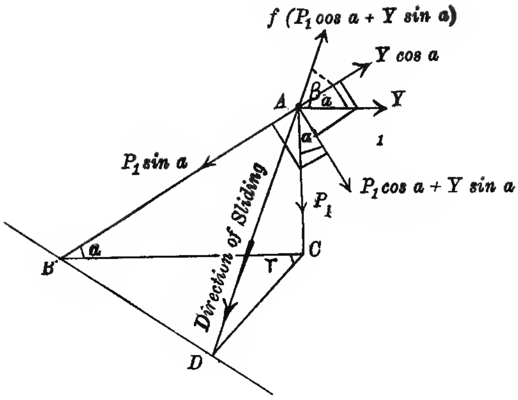


Fig. 18a.

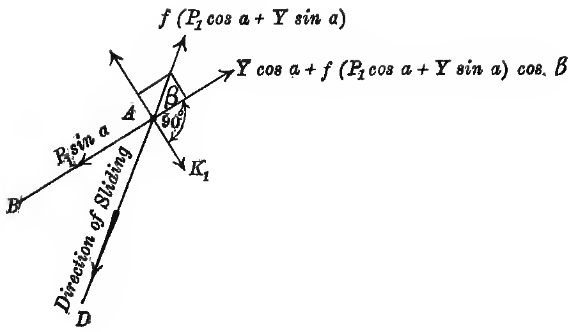


Fig. 19.

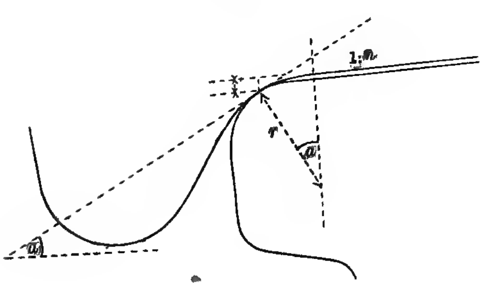
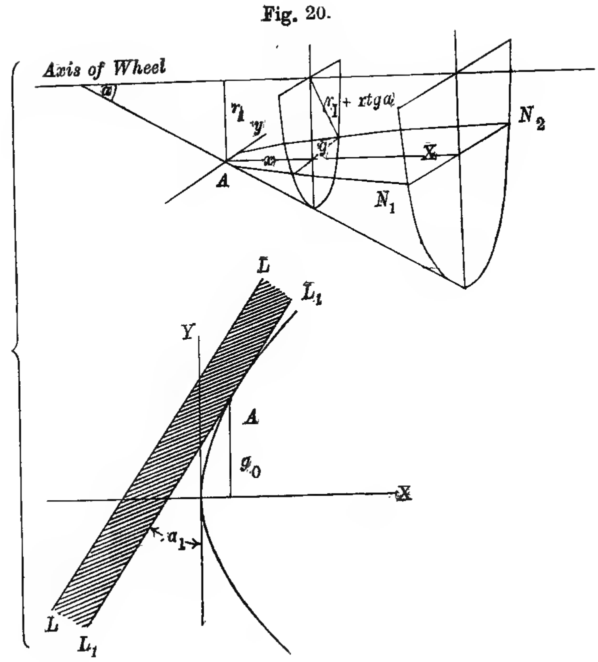


Fig. 20.



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§ 11.

The Position of the Point of Support at the Outer Fore-wheel relatively to the Wheel's Axis.

Because the leading F-wheel runs against the outer-rail at an angle α_1, [*i.e.* the so-called Angle of Approach] the middle point, A, of the surface of contact of this wheel lies at a distance y_0 in front of the vertical through the centre of the wheel; which distance is determined, from **Fig. 20,** as follows:—

Suppose the fairly flatly-rounded flange-hollow to be replaced at its point of contact with the rail by a conical-surfaced ring of which the cone-angle is α, and let this conical ring to be intersected in A by a horizontal plane parallel to the axle. Then we have for the intersection-line on the surface of the cone, the hyperbola $A\ N_1\ N_2$, of which the equation is

$$y^2 = 2\,\mathfrak{r}_1\,x \tan\alpha + x^2 \tan^2\alpha :$$

and on the surface of the rail-head, the straight lines $L\ L$ and $L_1\ L_1$, which latter is touched by the hyperbola in the point A, and forms an angle of $(90 - \alpha_1)$ with the same.
The co-ordinates of A are

$$y_0 = \frac{\mathfrak{r}_1 \tan\alpha_1}{\sqrt{\cot^2\alpha - \tan^2\alpha_1}}$$

$$x_0 = \mathfrak{r}_1\left(\frac{\cot^2\alpha}{\sqrt{\cot^2\alpha - \tan^2\alpha_1}} - \cot\alpha\right)$$

Since $\tan\alpha_1$ is generally very small compared with $\cot\alpha$ we have, approximately,

$$y_0 = \mathfrak{r}_1 \tan\alpha \tan\alpha_1 \qquad \ldots \qquad \ldots \qquad \ldots \qquad (22)$$

---

§ 12.

Practical Formulæ for the Pressure at the Fore-wheel.

With the assistance of Eqns. 15 and 16, the numerical values of Y for the above-mentioned vehicles can now be calculated as follows:—

The minimum value of $R\sigma$ which can come up for consideration corresponds to the minimum permissible† $R = 180^m$ and to a value of σ which is made up of the mean clearance $\cdot 0175^m$ allowed† in the straight plus that shown in Fig. 16 for $R = 180^m$, *i.e.*, $\cdot 02^m$.
Thus

$$R\sigma = 180\ (\cdot 0175 + \cdot 02) = 6\cdot 75^m.$$

For Passenger vehicles of 5^m wheel-base, when $R\sigma = 6\cdot 75^m$,

$$m = \frac{5}{2} + \frac{6\cdot 75}{5} = 3\cdot 85^m. \quad \text{(Eqn. 17).}$$

And if we put in Eqn. 19, $\quad f = \frac{1}{4} : *$
and *provisionally*, $\quad Y = 1\cdot 3\, f\, P_1 :$
and also $\quad \cos\beta = 1 :$

[† *i.e.* by the "Verein" Regulations or Standard Dimensions.—Trans.]

* Experiments carried out by Capt. Douglas Galton in 1878-79, mostly on the London-Brighton Ry. with the object of determining the amount of friction between brakes and wheels, and wheels and rails, at various velocities, gave the following coefficients of sliding friction between treads and rails.

Velocity per sec. in English feet. (Approx.)	f	
	Steel whe-l-tire and steel rail.	Steel wheel-tire, and iron rail.
88	·027	—
80	·038	—
70	·040	·060
60	·057	—
50	·065	·070
40	·070	—
20	·072	·073
10	·088	·095
just coming to rest	·242	·247

Vide "Engineering"— Vol. 25, pp. 469–472: Vol. 26, p. 386: Vol. 27, p. 371.

then $$\tan\alpha = \frac{1{\cdot}3.\,\frac{1}{4}.\,P_1 + \frac{1}{4}.P_1}{P_1 - \left(\frac{1}{4}\right)^2 1{\cdot}3.\,P_1} = {\cdot}626 = \tan 32^\circ\, 3',$$

and $\therefore \quad \alpha = 32^\circ\, 3'.$

On the assumption that the rail-head has a top-corner-rounding to a rad. $r = 12^m/_m$ and that $x_1 = {\cdot}485^m$, and $n = 20$; then

$$r\,(1 - \cos\alpha) = {\cdot}012\ (1 - \cos 32^\circ\, 3') = {\cdot}0018^m:$$

and (Equ. 21.) $$\frac{R\,\sigma}{n_1\,x_1} = \frac{6{\cdot}75}{20.\,{\cdot}485} + \frac{180}{{\cdot}485}{\cdot}0018$$

or $$= {\cdot}696 + {\cdot}667$$
$$= 1{\cdot}363^m.$$

Therefore we obtain for the angle β—Eqn. 20,

$$\cos\beta = \frac{3{\cdot}85}{\sqrt{3{\cdot}85^2 + \frac{1}{4}\,(1{\cdot}363 - 1{\cdot}5)^2}} = \frac{3{\cdot}85}{\sqrt{3{\cdot}85^2 + {\cdot}0049}} = 1.{\cdot}0\ldots$$

The initial *assumption* that $\cos\beta = 1$ was therefore correct; and the value thus obtained above of $\frac{R\sigma}{n_1\ x_1} = 1{\cdot}363^m$ may be used for the calculation of Y.

We can now determine from the general Eqn. 15 the share which the F. A. contributes to the horizontal pressure Y, viz., from the equation

$$Y_1 = f\,P_1 \frac{3{\cdot}85 - \frac{1{\cdot}5}{10}\,(1{\cdot}36 - 1{\cdot}5)}{\sqrt{3{\cdot}85^{\,2} + \frac{1}{4}\,(1{\cdot}36 - 1{\cdot}5)^{\,2}}};$$
$$= 1.\,f\,P.$$

For the H. A. *put* $r(1 - \cos\alpha) = {\cdot}001^m$; and we obtain

$$\frac{R\sigma}{n_2\,x_2} = \frac{6{\cdot}75}{20.\,{\cdot}485} + \frac{180}{{\cdot}485}{\cdot}001$$
$$= {\cdot}696 + {\cdot}392$$
$$= 1{\cdot}09^m.$$

Now $q = (d - m) = (5 - 3{\cdot}85) = 1{\cdot}15^m$. Whence, from Eqn. 15, the contribution by the H. A. is

$$Y_2 = f\,P_2 \frac{\frac{1{\cdot}5}{10}\,(1{\cdot}09 + 1{\cdot}5)}{\sqrt{1{\cdot}15^{\,2} + \frac{1}{4}\,(1{\cdot}09 + 1{\cdot}5)^2}} = {\cdot}224\,P_2$$

If $\quad P_1 = P_2$

then $$Y = Y_1 + Y_2 \quad \ldots \quad \ldots \quad \ldots \quad (23)$$
$$= 1.\,f\,P + {\cdot}224\,f\,P$$
$$= 1{\cdot}224\,f\,P:$$

and therefore, $\quad \alpha = 31^\circ\, 4'.$

On page 19 it is shown that the radial H. A. when $R\sigma = 39{\cdot}6$, contributes nothing to the value of Y. For this particular value of $R\,\sigma$, corresponding to $\sigma = {\cdot}0175^m$, and $R = 2263^m$, the value of Y is obtained as follows:—

Assume $\quad Y_1 = 1\,f\,P;\ \cos\beta = {\cdot}8;\ f \doteq {\cdot}25;$

then by Eqn. 19, $\quad \tan\alpha = {\cdot}5$, and $\therefore\ \cos\alpha = {\cdot}891.$

and by Eqn. 21 :— $\quad \frac{R\,\sigma}{n_1\,x_1} = 10{\cdot}24,$

Accordingly the exacter values are—(being radial, $m = d = 5^m$.) – Eqn. 20,

$$\cos\beta = \frac{5}{\sqrt{25 + \frac{1}{4}(10{\cdot}24 - 1{\cdot}5)^2}} = {\cdot}753$$

Whence $\beta = 41°40'$, $\tan \alpha = \cdot46$, $\alpha = 24° 40'$, $\dfrac{R\sigma}{n_1 r_1} = 9{\cdot}08$;

and
$$Y_1 = \frac{5 - \frac{1\cdot5}{10}(9\cdot08 - 1\cdot5)}{\sqrt{25 + \frac{1}{4}(9\cdot08 - 1\cdot5)^2}}$$
$$Y = Y_1 = \cdot62\, f P_1$$

For the value of $R\sigma$ below 39·6, if the H. A. actually stands radial, we have, for its contribution—Eqn. 16—

$$Y_2 = f P_2 \frac{1\cdot5}{5} = \cdot3 f P_2:$$

and therefore in this case, $Y = \cdot62\, f P_1 + \cdot3\, f P_2 = \cdot92\, f P$ ... ... (24)

From the above values of Y given by Eqns. 23 and 24, and on the assumption—which has been found experimentally to be sufficiently exact—that as R increases from 180^m up to 2263^m, Y decreases proportionately from $1{\cdot}22 f P$ to $\cdot92 f P$—we obtain the expression

$$Y = \left(1\cdot25 - \frac{R}{6800}\right) f P. \quad \ldots \quad \ldots \quad \ldots \quad (25)$$

In this way the values of α and formulæ for and values of Y exhibited in the following Table are obtained.

Vehicle.	d	r	$R\sigma$	σ	R	Y	α	formula for Y
	$s = 1{\cdot}5^m$					$r = 12^{mm}$: $n = 20$: $f = \frac{1}{4}$.		
	Metres							
Passenger cars...	5	·485	6·75	·0375	180	$1{\cdot}224 f P$	31°4′	$\left(1\cdot25 - \frac{R}{6800}\right) fP$
			39·60	·0175	2263	$\cdot92 f P$	24°40′	
Goods waggons..	4	·485	6·75	·0375	180	$1{\cdot}35 f P$	32°41′	$\left(1\cdot39 - \frac{R}{4200}\right) fP$
			30·6	·0175	1750	$\cdot98 f P$	24°54′	
Sec.-Line locos...	2·5	·535	6·75	·0375	180	$1{\cdot}60 f P$	35°50′	$\left(1\cdot69 - \frac{R}{2000}\right) fP$
			22·3	·0175	1280	$1{\cdot}05 f P$	25°5′	

For sharp curves we have accordingly the following Table :—

Vehicle.	d	$R\sigma = 6{\cdot}75$, $\sigma = \cdot0375$, $R = 180^m$. Y kg.	α	$R\sigma = 9$, $\sigma = \cdot03$, $R = 300^m$ Y kg.	α	$R\sigma = 12$, $\sigma = \cdot02$, $R = 600^m$ Y kg.	α
	$P = 5000$ kg.						
Passenger cars	5^m	1525	31° 4′	1510	30°52′	1460	30° 5′
Goods waggons	4^m	1690	32° 45′	1650	32°18′	1560	31° 20′
Sec.-Line locos	$2{\cdot}5^m$	200	35° 50′	1930	35° 6′	1840	33° 10′

These values show that Y and α for the same vehicle in sharp curves are almost constant.

§ 13.

The forces parallel to the axle at the Hind-wheels and Inner Fore-wheel.

The force G_1 acting at the tread of the inner F-wheel and parallel to its axles, tending to overturn the inner rail—is given by Eqn. 14, viz.

$$G_1 = f\,P \frac{m}{\sqrt{m^2 + \frac{1}{4}\left(\frac{R\sigma}{n_1 x_1} - s\right)^2}} \quad \ldots \quad \ldots \quad (26)$$

The following Table gives the values of this expression:

Vehicle.	d	G_1		
		$R=180^m$ $\sigma = \cdot 0375$	$R = 300^m$ $\sigma = \cdot 03$	$R = 1000^m$ $\sigma = \cdot 0175$
Passenger cars ...	5^m	$1.f\,P$	$1.f\,P$	$95.f\,P$
Goods waggons ...	4^m	$1.f\,P$	$1.f\,P$	$92.f\,P$
Sec.-Line locos ...	$2{\cdot}5^m$	$1.f\,P$	$1.f\,P$	$82.f\,P$

It now remains to determine what the action is of the H. A. wheels on the rails.

From the Fig. 15,

$$X\,d = -G_2\,d - (K_1 + K_2)s$$

$$\therefore\ X = f\,P_2 \frac{q + \frac{s}{2d}\left(\frac{R\sigma}{n_2 x_2} + s\right)}{\sqrt{q_2 + \frac{1}{4}\left(\frac{R\sigma}{n_2 x_2} + s\right)^2}} - f\,P_1 \frac{\frac{s}{2d}\left(\frac{R\sigma}{n_1 x_1} - s\right)}{\sqrt{m^2 + \frac{1}{4}\left(\frac{R\sigma}{n_1 x_1} - s\right)^2}} \quad \ldots \quad (27)$$

Further,

$$G_2 = -\,f\,P_2 \frac{q}{\sqrt{q^2 + \frac{1}{4}\left(\frac{R\sigma}{n_2 x_2} + s\right)^2}} \quad \ldots \quad \ldots \quad (28)$$

And when $q = o$,—viz. H. A. radial,

then

$$X = G_2 = \frac{Y - G_1}{2} \quad \ldots \quad \ldots \quad \ldots \quad (29)$$

From the Eqns. 27, 28, and 29, we have in the following Table, on the assumption that $P_1 = P_2 = P$, the values of X and G_2 shown therein.

Vehicle.	d	$R = 180^m$ $\sigma = \cdot 0375$		$R = 300^m$ $\sigma = \cdot 03$		$R = 1000^m$ $\sigma = \cdot 0175$	
		X	G_2	X	G_2	X	G_2
Passenger cars	5^m	$\cdot 88\,f\,P$	$-\cdot 66\,f\,P$	$\cdot 66\,f\,P$	$-45\,f\,P$	$\cdot 08\,f\,P$	$\cdot 08\,f\,P$
Goods waggons	4^m	$\cdot 60\,f\,P$	$-\cdot 25\,f\,P$	$76\,f\,P$	$-\cdot 16\,f\,P$	$\cdot 12\,f\,P$	$\cdot 12\,f\,P$
Sec.-Line locos	$2{\cdot}5^m$	$\cdot 30\,f\,P$	$-\cdot 30\,f\,P$	$\cdot 27\,f\,P$	$-\cdot 27\,f\,P$	$\cdot 16\,f\,P$	$\cdot 16\,f\,P$

§ 14.

The Curve-Pressure in the case of Braked Vehicles.

When all the wheels are firmly braked they skid, and the moments $K_1 s$, $K_2 s$, disappear; and therefore—see p. 18,

$$Y = G_1 = f P_1 \frac{\mathfrak{b}}{\sqrt{\mathfrak{b}^2 + c^2}},$$

where $\mathfrak{b} = \frac{Y}{R} d$: and $c = Y$:

therefore
$$Y = f P_1 \frac{\frac{d}{R}}{\sqrt{\left(\frac{d}{R}\right)^2 + 1}},$$

or, sufficiently accurately,

$$Y = f P_1 \frac{d}{R} \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (30)$$

From this we obtain the following Table:—

d	Y		
	$R = 180^m$	$R = 500^m$	$R = 1000^m$
5^m	$\frac{fP}{36}$	$\frac{fP}{100}$	$\frac{fP}{200}$
4^m	$\frac{fP}{45}$	$\frac{fP}{125}$	$\frac{fP}{250}$

These forces, in comparison with those occurring with rolling *i.e.* unbraked, wheels are quite insignificant; and consequently braked axles deviate very easily from their momentary directions of motion. This explains why braked axles so easily derail in curves.

---

§ 15.

The Influence of Curve-Radius on the Position of the Vehicle.

When $R\sigma \geqq \frac{d^2}{2}$, (see Eqn. 17*a*) then when $Y > G_1$ there is—Eqn. 29—a force acting in each H-wheel radially outwards; and in the reverse direction, when $Y < G$.

Since the H-wheels always slide longitudinally—unless it happen in some particular instance that $\rho = R$, and so $K_2 = 0$ — these wheels must have, in addition, a motion in the direction of their axle due to the forces X and G_2.

Consequently, in *sharp* curves, where $Y > G_1$, the vehicle rotates about a vertical axis lying some distance *behind* the H. A. if $R\sigma > \frac{d^2}{2}$; and in *flat* curves, where $Y < G_1$ the axis of rotation lies *between* the two axles.

In the previous discussion the vehicle's rotation-axis has, however, been assumed as lying in the H. A. It has been so assumed because a considerable simplification of the investigation is thus obtained; and for *sharp* curves, where we are mainly concerned with the determination of Y, G_1, X, and G_2, there arises therefrom no error worth mentioning. Only it is to be borne in mind that these forces are in general somewhat smaller than the values which we have found for them.

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§ 16.

Influence of the Tension in the Coupling on the Forces acting between Wheel and Rail.

In **Fig. 21**, let

l = the distance apart of the *front* buffer-beams of two consecutive vehicles.

$\epsilon = \frac{l}{R}$, the angle included between two curve-radii drawn to these vehicle-ends.

$\mathfrak{d}$ = the distance of the point of application of the coupling-chain, at the fore-end, from the vertical rotation-axis of the vehicle.

l_1 = the distance between the points of attachment of the conpling at the two ends of the vehicle.

g = the length of the coupling-chain from hook to hook.

γ = the axle included by the coupling with the fore buffer-beam.

Z = the tension in the Fore-coupling.

$Z\text{-}z$ = ditto ditto Hind-coupling.

The tractive forces Z, $Z\text{-}z$, which are inclined to each other at the angle ϵ reduce—see **Fig. 22.**—to a single force, k, and a couple, $(Z\text{-}z)\, l_1 \cos(\gamma + \epsilon)$.

Also from the Figure

$$k^2 = Z^2 + (Z\text{-}z)^2 - 2\, Z\,(Z\text{-}z) \cos \epsilon.$$

Putting $Z = nz$, where n = the number of vehicles over which Z is uniformly distributed; then

$$k^2 = z^2 \,[2\, n\,(n-1)\,(1 - \cos\epsilon) + 1].$$

The component in the direction of the vehicle's longitudinal axis, $k \sin(\gamma - \delta)$, maintains the forward motion of the vehicle on the rails; and the component parallel to the wheel axle, $k \cos(\gamma - \delta)$, produces its rotation, and at the same time modifies the magnitude of Y.

This component is deducible from

$$k \sin \delta = (Z\text{-}z) \sin \epsilon$$

assuming $\sin \epsilon = \frac{l}{R}$—sufficiently approximate—and so after several intermediate steps we obtain

$$k \cos(\gamma - \delta) = z \left[\cos\gamma \sqrt{1 - \frac{l^2}{R^2}(n-1)\left(\frac{n}{2} - 1\right)} + \sin\gamma\,(n-1)\,\frac{l}{R}\right] \quad \ldots \quad (31)$$

The decrease in Y due to the tractive force $Z\text{-}z$ and to $k \cos(\gamma - \delta)$ is

$$\mathfrak{y} = \frac{z\,(n-1)\, l_1 \cos(\gamma + \epsilon) + k \cos(\gamma - \delta)\, \dfrac{l_1 + d}{2}}{d} \quad \ldots \quad \ldots \quad (32)$$

We will now prove that the tension in the coupling has the direction indicated in Fig. 22, and that the expression for it—Eqn. 32—is positive.

If, in the **Fig. 23**, $C\,D$ be the axis of the fore-vehicle, and $A\,B$ that of the hinder, a_1 the point of intersection of both vehicle axes, and b the point of attachment of the coupling at the hind-end of the fore-vehicle, then

$$\gamma + \epsilon \lesseqgtr 90$$

according as

$$\left.\begin{array}{r} b \text{ and } a_2 \\ \text{or, } b \text{ and } a_1 \\ \text{or, } b \text{ and } a \end{array}\right\} \text{are the two points of attachment respectively.}$$

Consequently,

$$\gamma + \epsilon \lesseqgtr 90^\circ, \text{ according as } \mathfrak{d} - \frac{l}{2} \gtreqless 0.$$

Fig. 21.

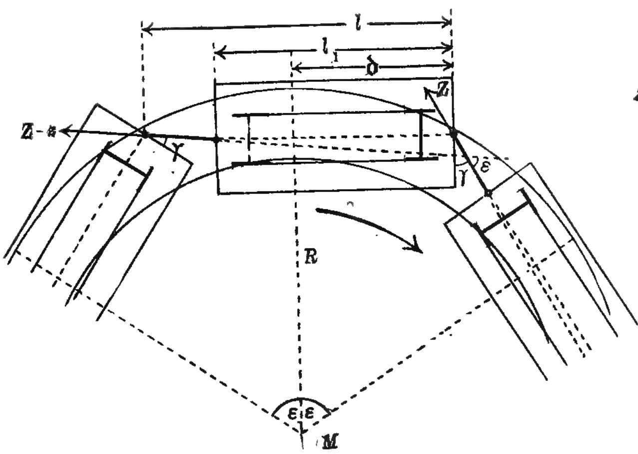


Fig. 22.

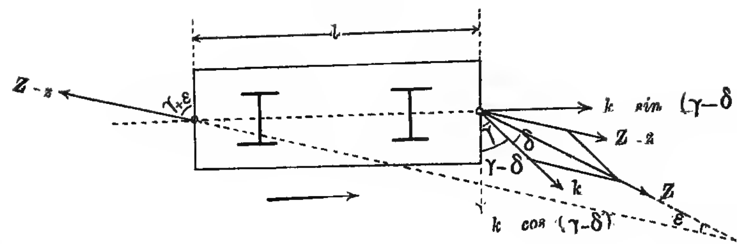


Fig. 23.

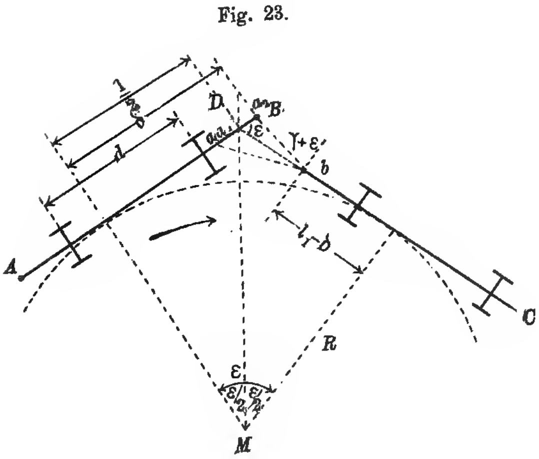
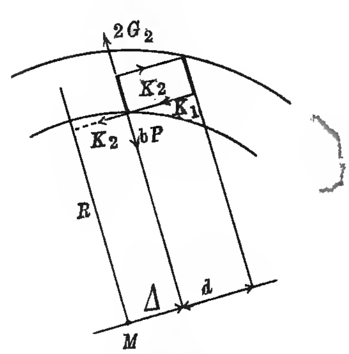


Fig. 24.



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For the Passenger and Goods vehicles in use on the Prussian State Railways the following Table shows that in curves of 180^m, $(\mathfrak{v} - \frac{l}{2})$ has the values $\cdot 925^m$ and $1\cdot 265^m$, and is therefore positive.

d	l_1	g	l	$R = 180^m$		
				m	$\mathfrak{v}$	$\mathfrak{v} - \frac{l}{2}$
Metres						
5	8·85	·85	9·7	3·85	5·83	·925
4	7·65	·85	8·5	3·69	5·52	1·265

Since now $\mathfrak{v}$ and R increase together, $(\gamma + \epsilon)$ is always less than 90° in the case of the above vehicles under ordinary conditions; and thus these vehicles when running in a train are drawn by the fore-end coupling *inwards*, and by that at the hind-end, *outwards*. From Fig. 23, in order to transform Eqn. 32, we have

$$\cos(\gamma + \epsilon) = \frac{a_1\, a_2}{a_2\, b} \sin \epsilon = \frac{\mathfrak{v} - \frac{l}{2}}{g} \cdot \frac{l}{R}$$

$$\cos \gamma = \frac{a_1\, b}{a_2\, b} \sin \epsilon = \frac{g + 2\,\mathfrak{v} - l_1}{2\, g} \cdot \frac{l}{R}$$

$$\sin \delta = \frac{Z\text{-}z}{k} \sin \epsilon = \frac{z\,(n - l)}{k} \cdot \frac{l}{R}$$

$$k^2 = z^2 \left[1 + n\,(n-1)\frac{l^2}{R^2}\right]:$$

and therefore,

$$k.\cos(\gamma - \delta) = z\frac{l}{R}\left[\frac{g + 2\,\mathfrak{v} - l_1}{2\, g} + (n-1)\right]:$$

likewise

$$z\,(n-1)\, l_1 \cos(\gamma + \epsilon) = z\,(n-1)\, l_1 \frac{\mathfrak{v} - \frac{l}{2}}{g} \frac{l}{R}\,.$$

And finally, substituting the above in Eqn. (32) we have

$$\mathfrak{y} = \frac{z\, l}{R\, d}\left[\frac{g + 2\mathfrak{v} - l_1}{4\, g}\,(l_1 + d) + (n-1)\left(\frac{\mathfrak{v} - \frac{l}{2}}{g}\, l_1 + \frac{l_1 + \mathfrak{v}}{2}\right)\right] \quad \dots \quad (33)$$

For the outwards-acting force in the H. A., due to the tension in the coupling, we have

$$\mathfrak{y}_1 = \frac{z\, l}{R\, d}\left[\frac{g + 2\mathfrak{v} - l_1}{4\, g}\,(l_1 - d) + (n-1)\left(\frac{\mathfrak{v} - \frac{l}{2}}{g}\, l_1 + \frac{l - d}{2}\right)\right] \quad \dots \quad (34)$$

From Eqns. 33 and 34, for loaded goods waggons of 20,000 *kg.* gross weight, we have in the following Table the values of $\mathfrak{y}$ and $\mathfrak{y}_1$ in the nth vehicle, if we substitute for $R = 180^m$, $\mathfrak{v} = 5\cdot 515^m$, the corresponding $R\sigma = 6\cdot 75$, and $z = 7 \times 20 = 140$ *kg.*

$R = 300^m$, $\mathfrak{v} = 5\cdot 825^m$, the ,, $R\sigma \underset{>}{=} \frac{4^2}{2}$, and $z = 5 \times 20 = 100$ *kg.*

$R = 600^m$, $\mathfrak{v} = 5\cdot 825^m$, the ,, $R\sigma \underset{>}{=} \frac{4^2}{2}$, and $z = 3\cdot 5 \times 20 = 70$ *kg.*

$d = 4^m$, $l = 8\cdot 5^m$, $l_1 = 7\cdot 65^m$, $g = \cdot 85^m$.			
R m	$R\,\sigma$ m	$\mathfrak{y}$ *kg.*	$\mathfrak{y}_1$ *kg.*
180	$= 6\cdot 75$	$26 + (n-1)\, 32$	$8\cdot 4 + (n-1)\, 24\cdot 6$
300	$\underset{>}{=} \frac{d^2}{2}$	$8\cdot 6 + (n-1)\, 17$	$3\cdot 6 + (n-1)\, 11\cdot 2$
600	$\underset{>}{=} \frac{d^2}{2}$	$3 + (n-1)\, 5$	$1\cdot 3 + (n-1)\, 4$

From these Formulæ for $\mathfrak{y}$ and $\mathfrak{y}_1$ the figures in the following Table are obtained for the 1st, 10th, 20th, and 30th vehicles in a train in curves of 180^m, 300^m, and 600^m. Here n is counted from the end of the train; the load on each axle is 10,000 $kg.$ and the total resistance per tonne is, as above

$$\begin{aligned} 7\ kg.\ &\text{for } R = 180^m \\ 5\ kg.\ &\text{,, } R = 300^m \\ 3{\cdot}5\ kg.\ &\text{,, } R = 600^m \end{aligned}$$

$d = 4^m$. $4P = 20{,}000\ kg.$						
n	$R = 180^m,\ R\sigma = 6{\cdot}75$		$R = 300^m,\ R\sigma \gtreqless \frac{d^2}{2}$		$R = 600^m,\ R\sigma \gtreqless \frac{d^2}{2}$	
	$\mathfrak{y}$	$\mathfrak{y}_1$	$\mathfrak{y}$	$\mathfrak{y}_1$	$\mathfrak{y}$	$\mathfrak{y}_1$
	Kgs.					
1	26	8·4	8·6	3·6	3	1·3
10	314	230	162	104	48	37
20	634	476	332	216	98	77
30	964	722	502	328	148	117

Accordingly, the tension in the coupling diminishes the values of Y—obtained in §12, [page 23, lower Table] — for $d = 4^m$, and $P = 5000\ kg$, from the last vehicle, up to the 30th as follows :—

in curves of $R = 180^m$, from $(1690 - 26) = 1664\ kg.$ to $(1690 - 964) = 726\ kg.$
.... $= 300^m$, ... $(1650 - 8{\cdot}6) = 1641\ kg.$ to $(1650 - 502) = 1148\ kg.$
.... $= 600^m$, ... $(1560 - 3) = 1557\ kg.$ to $(1560 - 148) = 1412\ kg.$

The pressure $Y - \mathfrak{y}$—acting between the outer F-wheel and the outer-rail varies from the last vehicle to the 30th vehicle

in curves of 180^m R by $(964 - 26) = 838\ kg$, or by 50% of Y.
300^m R ,, $(502 - 8{\cdot}6) = 493\ kg$, ,, ,, 30% of Y.
600^m R ,, $(148 - 3) = 145\ kg$, ,, ,, 9% of Y.

§ 17.

The influence of Tension in the Couplings on the Position of the Vehicle.

On the assumption that $\mathfrak{y}_1 = 0$, it was found from Eqn. 29 that when a goods waggon traversed a curve of 180^m the inner H-wheel exerted a force $X = {\cdot}6fP$ on the rail. (see Table at end of § 13).

Suppose the vehicle so far from the end of the train that $\mathfrak{y}_1 = {\cdot}6fP = {\cdot}6 \,.\, \frac{1}{4} .\, 5000 = 750\ kg$; then the inner H-wheel has only itself to displace. From the formula for $\mathfrak{y}_1$ given in the preceding Table, this will occur when $n = \dfrac{750 - 8{\cdot}4}{24{\cdot}6} + 1 = 31$st vehicle, and in each following vehicle up to the end one $\mathfrak{y}_1$ will therefore be $> 750\ kg$: whence it follows, that in these vehicles a part of $\mathfrak{y}_1$ is employed in the displacement of the inner H-wheel. $\mathfrak{y}_1$ has no influence on the position of these vehicles in the curve; that could only happen if $\mathfrak{y}_1$ became sufficiently large to slide not only the outer, but also the inner H-wheel. But this latter may easily happen if the waggon is *empty and runs in a train of loaded vehicles.*

When a vehicle runs in a curve of 300^m and $R\sigma = 9^m$ then, on the assumption made in the previous discussion, § 13, that $\mathfrak{g}_1 = 0$, there must be a force $(X + G_2) = \cdot 16\, f P + \cdot 16\, f P = \cdot 32\, f P = \cdot 32\, \frac{1}{4}\, 5000 = 400$ *kg* exercised by the rail on the hind-wheels towards the opposite or convex side of the curve in order to maintain the H. A. in the radial position. But this is only possible in the special case where the H-wheels do not slide in the longitudinal direction, namely, for true conical motion, where the apex of the hind-axle rolling-cone coincides with the curve-centre.

Whence it follows that in the curve of 300^m where $\mathfrak{g}_1$ only reaches the value of of 400 *kg* at the $n^{th} = \frac{400 - 3{\cdot}6}{11{\cdot}2} + 1 = 45^{st}$ vehicle, the last 44 vehicles of the goods-train rotate about a vertical axis lying behind the H. A.: whereas for the 45th waggon, the rotation-axis lies in the H. A.; and before the H. A. in all the after-succeeding ones, (46th, 47th, &c).

Similarly, for curves of 600^m, the vertical rotation-axis for $n = 1$ up to 48 lies behind the H. A.; and for $n > 48$, in front of it.

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§ 18.

Influence of abnormal Superelevation of Outer-Rail on the Position of the Vehicle.

In § 17 it has been shown that the last 44 waggons in a loaded goods-train when running through a curve of 300^m, and likewise the last 48 vehicles in their passage through a curve of 600^m *R*, rotate about a vertical axis lying behind the H. A.—if the elevation of the outer-rail exactly corresponds to the height due to the velocity. If the velocity is below the amount corresponding to the actual superelevation if, for example, $V = 6^m$, while the V, corresponding to the superelevation h, is 17·12^m, (*i.e.*, corresponding to the formula $h = \frac{45}{R}$), then in each axle there is a centripetal force of $\frac{P}{g} \cdot \frac{17{\cdot}12^2 - 6^2}{R} = \frac{52}{R} P$, say, or $= \cdot 173 P$, if $R = 300^m$; and $= \cdot 087\, P$, when $R = 600^m$.

This force compels the H. A. to move closer to the inner-rail, and so causes the rotation-axis to move still further away from the F. A.

This rotation-axis lies at its maximum distance behind the H. A. in the *last* or end-vehicle of the train.

Call this distance behind the H. A. Δ, and the force exerted on the H. A. due to abnormal superelevation $b\, P_2$; then with *cylindric* treads—see **Fig. 24**—we have ($q = 0$, $X = G_2$—vide Eqn. 29.)

$$s\,(K_1 + K_2) - b P_2 d + 2\, G_2\, d = 0:$$

in which—from Eqns. 13, 14—

$$K_1 = -f P_1 \frac{\frac{s}{2}}{\sqrt{(d + \Delta)^2 + \frac{s^2}{4}}}$$

$$K_2 = -f P_2 \frac{\frac{s}{2}}{\sqrt{\Delta^2 + \frac{s^2}{4}}}$$

$$G_2 = f P_2 \frac{\Delta}{\sqrt{\Delta^2 + \frac{s^2}{4}}}.$$

Putting in addition $P_1 = P_2$

then

$$\frac{f\,(2\,d\,\Delta - \frac{s^2}{2})}{\sqrt{\Delta^2 + \frac{s^2}{4}}} = b\,d + \frac{f\,\frac{s^2}{4}}{\sqrt{(d+\Delta)^2 + \frac{s^2}{4}}}$$

and for $s = 1{\cdot}5^m$, $d = 4^m$, $f = \frac{1}{4}$, we have

$$\frac{2\,\Delta - {\cdot}281}{\sqrt{\Delta^2 + {\cdot}5625}} = 4\,b + \frac{{\cdot}281}{\sqrt{(4+\Delta)^2 + {\cdot}5625}}.$$

Whence, for $R = 300^m$, and $b = {\cdot}173$, $\Delta = {\cdot}5^m$.
$R = 600^m$, $b = {\cdot}087$, $\Delta = {\cdot}3^m$.

The maximal value that Δ can assume is given by the expression—obtained from Fig. 24—

$$\left(\frac{d + \Delta_{\max}}{2\,R}\right)^2 = \sigma + \frac{\Delta^2_{\max}}{2R}$$

and for

$$R = 300^m,\quad R\,\sigma = 8{\cdot}25^m, \text{ and } d = 4^m:\ \Delta_{\max} = {\cdot}1125^m$$

and for

$$R = 600^m,\quad R\,\sigma = 12^m,\qquad d = 4^m:\ \Delta_{\max} = 1{\cdot}0^m.$$

Consequently, in a curve of 300^m R, when $d = 4^m$ and $R\,\sigma = 8{\cdot}25^m$, Δ cannot amount to ${\cdot}5^m$, but must be at most ${\cdot}1125^m$. Accordingly, it is not only at the last vehicle in the train that the H. A. runs closely against the inner-rail, but also in a large number of vehicles further ahead.

In the curve of 300^m, Δ_{max} is greater by $(1 - {\cdot}3) = {\cdot}7$ than the value found above of Δ; and therefore the inner H-wheel runs against the inner-rail in this curve only when other forces co-operate with $b\,P$. The pressure of the wind, for example, acting in the same direction as $b\,P$, might drive the H-wheels towards the inner-rail.

When the superelevation is normal, $b = 0$; then for $d = 4^m$, and from the above equation, we have

$$\frac{2\,\Delta - {\cdot}281}{\sqrt{\Delta^2 + {\cdot}5625}} = \frac{{\cdot}281}{\sqrt{(d+\Delta)^2 + {\cdot}5625}}\ ,$$

Whence always $2\,\Delta > {\cdot}281^m$,

or $\Delta > {\cdot}140^m$.

Thus in the normally superelevated curve of 300^m R, $\Delta > \Delta_{\max}$; whence in this curve also, the inner H-wheel of the last or end-vehicle runs close-up against the inner-rail.

On the other hand, if the outer-rail be too little superelevated then, after the preceding discussion, it is only necessary to remark that $b\,P$ presses the H. A. towards the outer-rail, and displaces the rotation-axis further towards the F. A.

When the treads are not cylindric, but coned, the above conditions are but very slightly altered.

The influence of f on the position of the last vehicle has not been taken into consideration. From the equation for Δ it follows that with a normal superelevation of the outer-rail, f has no effect on Δ; and that with abnormal superelevation Δ increases under the influence of $b\,P$ when f decreases; and decreases if f increases.

§ 19.

The Resistance of a Vehicle due to Curvature.

According to Eqn. 22 the point of contact of the guiding F-wheel lies at a distance $y_o = x_1 \tan\alpha \tan\alpha_1$ in front of a vertical through the centre of the wheel. The wheel resists, consequently, the forward movement of the vehicle with a moment of $P\,x_1 \tan\alpha \tan\alpha_1$: or, when referred to the vehicle's axis, with a force of $P.\tan\alpha\tan\alpha_1$, and gives thus a *coefficient* of resistance

$$W_1 = \frac{P.\tan\alpha\tan\alpha_1}{4P} = \frac{\tan\alpha\tan\alpha_1}{4}.$$

In addition, there is at the F. A. a resistance due to the action of K_1, which is not removed by the downsliding of the fore-wheel on the rounded corner of the rail-head; and the work done against this resistance is

$$K_1 . \Delta s = K_1\, 2\, c_1 - \text{per second}:$$

giving a coefficient of resistance,

$$W_2 = \frac{K_1\, 2c_1}{\Delta\, 4P}.$$

Similarly, for the H. A.

$$W_3 = \frac{\tan\alpha' \tan\alpha'_1}{4},$$

and

$$W_4 = \frac{K_2\, 2c_2}{\Delta\, 4P},$$

where α', α'_1, K_2, and c_2, are the corresponding H. A. values of α, α_1, K_1, and c_1. Consequently, we can now establish a formula, with the assistance of the expressions for Y, K_1, &c., which will express the increased resistance to motion of 2-axled vehicles in curves over that in straights, viz. the Curve-resistance.

We have generally, Eqn. 25,

$$Y = (a_1 - b_1 R)\, f\, P,$$

where a_1 and b_1 are definite coefficients depending on the type of vehicle: and for sharp curves, $\cos\beta = 1$, is sufficiently exact.
Accordingly, from Eqn. 19,

$$\tan\alpha = \frac{\frac{Y}{P} + f}{1 - \frac{Y}{P}f} = \frac{(a_1 + 1)\, f - b_1\, f\, R}{1 - a_1\, f^2 - b_1 f^2 R}.$$

Putting, generally, $\tan\alpha_1 = \frac{d}{R}$, we have

$$W_1 = \frac{1}{4} \cdot \frac{(a_1 + 1)\, f - b_1 f R}{1 - a_1 f^2 - b_1 f^2 R} \cdot \frac{d}{R}.$$

Also, putting for K_1 the value $f\,P$ instead of that given by Eqn. 13, and for c_1 the following expression corresponding to $\rho_1 > R$, viz.

$$c_1 = \frac{V}{2}\left(\frac{s}{R} - \frac{\sigma}{n_1 x_1}\right),$$

we obtain

$$W_2 = \frac{f}{4}\left(\frac{s}{R} - \frac{\sigma}{n_1 x_1}\right).$$

For the H. A, neglecting W_3 for sharp curves where $R\sigma < \frac{d^2}{2}$,

$$W_4 = \frac{f}{4}\left(\frac{s}{R} + \frac{d^2}{n_2 x_2 R} - \frac{\sigma}{n_2 x_2}\right)$$

Further, putting $n_1 x_1 = n_2 x_2$,
and since

$$W = W_1 + W_2 + W_4.$$

we obtain

$$W = \frac{f}{4}\left[\frac{d}{R} \cdot \frac{a_1 + 1 - b_1 R}{1 - a_1 f^2 + b_1 f^2 R} + \frac{2s + \frac{d^2}{nx}}{R} - \frac{2\sigma}{nx}.\right]$$

For $n\,x = 10, f = \frac{1}{4}$, and $s = 1{\cdot}5^m$, we thus obtain

$$W = \frac{d}{R} \cdot \frac{a_1 + 1 - b_1 R}{16 - a_1 + b_1 R} + \frac{3 + {\cdot}1\, d^2}{16\, R} - \frac{\sigma}{80} \quad \ldots \quad \ldots \quad (35)$$

This equation holds for curves in which $\frac{s}{R} \geqq \frac{\sigma}{n\,x}$; or $R\,\sigma \leqq s\,n\,x$, and thus also, on our assumptions, for $R\,\sigma \leqq 15$.

For $d = 4^m$, substitute values from the first Table of § 12: $a_1 = 1{\cdot}39$, $b_1 = \frac{1}{4200}$:

and thus obtain

$$W = \frac{58344 - 3{\cdot}7125\, R}{61946{\cdot}4\, R + R^2} - \frac{\sigma}{80} \quad \ldots \quad \ldots \quad (36)$$

For $d = 5^m$, $a_1 = 1{\cdot}25$, and $b_1 = \frac{1}{6800}$, we obtain

$$W = \frac{115629 - 4{\cdot}656\, R}{104504\, R + R^2} - \frac{\sigma}{80}. \quad \ldots \quad \ldots \quad (37)$$

From this equation, taking the mean values of σ derived from Fig. 16, we obtain the figures given in the following Table:—

R m	σ mm	W $d = 4^m$	$d = 5^m$
180	37·5	·004890	·005615
300	27·5	·002719	·003295
600	20	·001246	·001539
800	17·5	·000882	·001109

With the object of more clearly displaying how the curve-resistance increases with the *wheel-base*, in the following Table are given the values in kilogrammes for a gross-load of 20 tonnes.

R	Resistance in kgs. for $4\,P = 20\,t$. $d = 4^m$	$d = 5^m$	Difference due to wheel-base *kg.*
180^m	97·80	112·30	14·50
300^m	54·38	65·90	11·52
600^m	24·92	30·78	5·86
800^m	17·64	22·18	4·54

The errors involved in the assumptions made when obtaining Eqn. 35, viz., that $K_1 = fP$, $n_1 x_1 = n_2 x_2 = 10$, and in sharp curves, $W_3 = 0$, neutralize each other, and besides, are quite insignificant.

The method we have employed in the above investigation of Curve-resistance is one employed by the Author in a former work of his on the same subject.* It may possibly appear not sufficiently satisfactory to those who have not the leisure to balance one with another the effects of the assumptions made therein on the final result and so to convince themselves of their permissibility.

We add therefore, the following *perfectly general investigation* in which no such assumptions are made.

In Eqn. 15, put $\frac{R\,\sigma}{n_1 x_1} - s = a$, and $\frac{R\,\sigma}{n_2 x_2} + s = b$:

then for the Fore Axle,

$$Y_a = fP \left[\frac{m - \frac{s\,a}{2\,d}}{\sqrt{m^2 + \frac{a^2}{4}}} + \frac{s}{d} \cdot \frac{\frac{b}{2}}{\sqrt{q^2 + \frac{b^2}{4}}} \right].$$

* "Ueber die Bewegung vierrädriger Eisenbahnwagen in Curven."-Zeitschrift für Bauwesen, Jahrg XXIII S. 345 bis 388.

Further, from Eqns. 19 and 20: we have

$$\tan\alpha = f\frac{\sqrt{\dfrac{2m-\dfrac{sa}{2d}}{m^2+\dfrac{a^2}{4}}}+\sqrt{\dfrac{\dfrac{sb}{2d}}{q^2+\dfrac{b^2}{4}}}}{1-f^2\left[\sqrt{\dfrac{m-\dfrac{sa}{2d}}{m^2+\dfrac{a^2}{4}}}+\sqrt{\dfrac{sb}{q^2+\dfrac{b^2}{4}}}\right]\sqrt{\dfrac{m}{m^2+\dfrac{a^2}{4}}}}$$

Now

$$W_1 = \frac{\tan\alpha\tan\alpha_1}{4},$$

$$= \frac{f}{4}\frac{m}{R}\frac{\left(2m-\dfrac{sa}{2d}\right)\sqrt{m^2+\dfrac{a^2}{4}}+\dfrac{sb}{2d}\sqrt{\dfrac{m^2+\dfrac{a^2}{4}}{q^2+\dfrac{b^2}{4}}}}{m^2(1-f^2)+\dfrac{a^2}{4}+f^2\dfrac{sam}{2d}-f^2\dfrac{sbm}{2d}\sqrt{\dfrac{m^2+\dfrac{a^2}{4}}{q^2+\dfrac{b^2}{4}}}}.$$

And if $q=0$,

$$W_1 = \frac{f}{4}\frac{d}{R}\frac{\left(2d-\dfrac{sa}{2d}\right)\sqrt{d^2+\dfrac{a^2}{4}}+sd+\dfrac{sa^2}{4d}}{d^2(1-f^2)+\dfrac{a^2}{4}+f^2\dfrac{sa}{2}-f^2s\sqrt{d^2+\dfrac{a^2}{4}}}.$$

Again,

$$W_2 = \frac{f}{4}\sqrt{\frac{\dfrac{1}{2}\left(\dfrac{R\sigma}{n_1x_1}-s\right)}{m^2+\dfrac{1}{4}\left(\dfrac{R\sigma}{n_1x_1}-s\right)}}\left(\frac{\sigma}{n_1x_1}-\frac{s}{R}\right)$$

or

$$W_2 = \frac{f}{4R}\sqrt{\frac{\dfrac{a^2}{2}}{m^2+\dfrac{a^2}{4}}}.$$

For the Hind Axle we have:

$$X = fP\left[\sqrt{\frac{\dfrac{sb}{2d}}{q^2+\dfrac{b^2}{4}}}-\sqrt{\frac{\dfrac{sa}{2d}}{m^2+\dfrac{a^2}{4}}}\right]:$$

$$\tan\alpha = f\frac{2q+\dfrac{sb}{2d}\sqrt{q^2+\dfrac{b^2}{4}}-\dfrac{sa}{2d}\sqrt{\dfrac{q^2+\dfrac{b^2}{4}}{m^2+\dfrac{a^2}{4}}}}{q^2(1-f^2)+\dfrac{b^2}{4}-f^2\dfrac{sb}{2d}q+f^2\dfrac{sa}{2d}q\sqrt{\dfrac{q^2+\dfrac{b^2}{4}}{m^2+\dfrac{a^2}{4}}}}$$

$$\tan\alpha_1 = \frac{q}{R}:$$

consequently:

$$W_3 = \frac{f}{4}\frac{q}{R}\frac{\left(2q+\dfrac{sb}{2d}\right)\sqrt{q^2+\dfrac{b^2}{4}}-\dfrac{sa}{2d}\sqrt{\dfrac{q^2+\dfrac{b^2}{4}}{m^2+\dfrac{a^2}{4}}}}{q^2(1-f^2)+\dfrac{b^2}{4}-f^2\dfrac{sbq}{2d}+f^2\dfrac{saq}{2d}\sqrt{\dfrac{q^2+\dfrac{b^2}{4}}{m^2+\dfrac{a^2}{4}}}}$$

Again:

$$W_4 = \frac{f}{4R} \frac{\frac{b}{2}}{\sqrt{q^2 + \frac{b^2}{4}}}.$$

Accordingly,

$$W = W_1 + W_2 + W_3 + W_4$$

or quite generally,

$$W = \frac{f}{4R}\left[\frac{\left(2m^2 - \frac{sam}{2d} + \frac{sbm}{2d}\sqrt{\frac{m^2+\frac{a^2}{4}}{q^2+\frac{b^2}{4}}}\right)\sqrt{m^2+\frac{a^2}{4}}}{m^2(1-f^2) + \frac{a^2}{4} + f^2\frac{sam}{2d} - f^2\frac{sbm}{2d}\sqrt{\frac{m^2+\frac{a^2}{4}}{q^2+\frac{b^2}{4}}}} + \frac{\frac{a^2}{2}}{\sqrt{m^2+\frac{a^2}{4}}} + \right.$$

$$\left. + \frac{\left(2q^2 + \frac{sbq}{2d} - \frac{saq}{2d}\sqrt{\frac{q^2+\frac{b^2}{4}}{m^2+\frac{a^2}{4}}}\right)\sqrt{q^2+\frac{b^2}{4}}}{q^2(1-f^2) + \frac{b^2}{4} - f^2\frac{sbq}{2d} + f^2\frac{saq}{2d}\sqrt{\frac{q^2+\frac{b^2}{4}}{m^2+\frac{a^2}{4}}}} + \frac{\frac{b^2}{2}}{\sqrt{q^2+\frac{b^2}{4}}}\right] \quad \ldots \quad (38)$$

Whence, for

$$d = 4^m \; s = 1{\cdot}5^m \text{ and } f = \tfrac{1}{4},$$

We obtain finally the following equation:

$$W = \frac{1}{16R}\left[\frac{\left(2m^2 - \frac{1{\cdot}5\,am}{8} + \frac{1{\cdot}5\,bm}{8}\sqrt{\frac{m^2+\frac{a^2}{4}}{q^2+\frac{b^2}{4}}}\right)\sqrt{m^2+\frac{a^2}{4}}}{\frac{15}{16}m^2 + \frac{a^2}{4} + \frac{1{\cdot}5\,am}{128} - \frac{1{\cdot}5\,bm}{128}\sqrt{\frac{m^2+\frac{a^2}{4}}{q^2+\frac{b^2}{4}}}} + \frac{\frac{a^2}{4}}{\sqrt{m^2+\frac{a^2}{4}}} + \right.$$

$$\left. + \frac{\left(2q^2 + \frac{1{\cdot}5\,bq}{8} - \frac{1{\cdot}5\,aq}{8}\sqrt{\frac{q^2+\frac{b^2}{4}}{m^2+\frac{a^2}{4}}}\right)\sqrt{q^2+\frac{b^2}{4}}}{\frac{15}{16}q^2 + \frac{b^2}{4} - \frac{1{\cdot}5\,bq}{128} + \frac{1{\cdot}5\,aq}{128}\sqrt{\frac{q^2+\frac{b^2}{4}}{m^2+\frac{a^2}{4}}}} + \frac{\frac{b^2}{2}}{\sqrt{q^2+\frac{b^2}{4}}}\right] \quad \ldots \quad (39)$$

And if $q = o$, i.e. $m = d$, then

$$W = \frac{1}{16R}\left[\frac{(32 - {\cdot}75a)\sqrt{16+\frac{a^2}{4}} + 24 + \frac{1{\cdot}5}{4}a^2}{15 + \frac{a^2}{4} - \frac{1{\cdot}5\,a}{32} - \frac{15}{16}\sqrt{16+\frac{a^2}{4}}} + \frac{\frac{a^2}{2}}{\sqrt{16+\frac{a^2}{4}}} + b\right] \quad \ldots \quad (40)$$

The values of a and b to be substituted in Eqn. 39 are obtained from

$$a = \frac{R\sigma}{n_1 x_1} - s,$$

and

$$b = \frac{R\sigma}{n_2 x_2} + s.$$

But for Eqn. 40, only $a = \frac{R\sigma}{n_1 x_1} - s$, holds; and b is to be determined from the expression $\frac{R\sigma}{n_2 x_2} + s$ by replacing σ by *twice* the displacement of the H. A. relatively to the centre of track.

Fig. 25.

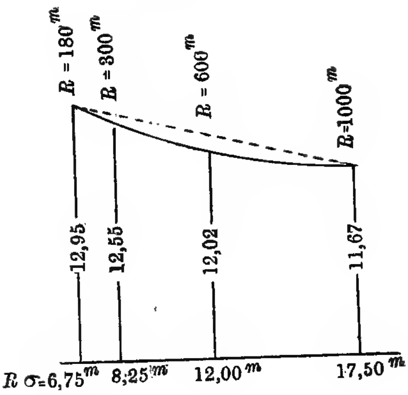
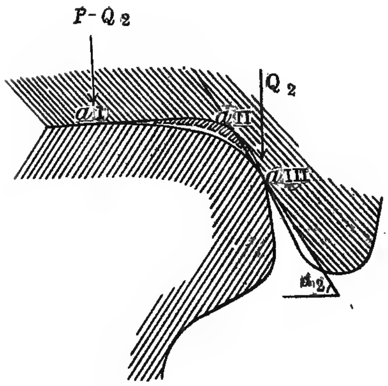


Fig. 26.



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Whence we obtain,

$$b = R\frac{\left(2\frac{d^2}{2R} - \sigma\right)}{n_2 x_2} + s = \frac{d^2 - R\sigma}{n_2 x_2} + s.$$

And since $d = 4^m$, $s = 1{\cdot}5^m$, $n_2 x_2 = 10$,

$$\therefore \quad b = \frac{16 - R\sigma}{10} + 1{\cdot}5.$$

Replacing the factor in brackets of Eqn. 39 by A, and the several individual terms in it by A_1, A_2, A_3, A_4; then for $R\sigma < \frac{d^2}{2}$,

$$W = \frac{A}{16R} = \frac{1}{16R}(A_1 + A_2 + A_3 + A_4) \quad \ldots \quad (41)$$

and for $R\sigma = > \frac{d^2}{16R}$,

$$W = \frac{1}{16R}(A_1 + A_2 + 0 + A_4) \ldots \quad \ldots \quad \ldots \quad (41a)$$

If with the assistance of the earlier found expressions for Y, and on the now familiar assumption that $x = 12^{mm}$, and $x_1 = x_2 = {\cdot}485^m$, the values of $n_1, x_1, n_2 \cdot x_2, q, m, a,$ and b are calculated for the curve radii and the respective clearances σ given in the following Table, and inserted in the expressions for A_1, A_2, A_3, and A_4, and the results summed for the total A, then we obtain the quantities shewn in the respective columns of the following Table:—

$d = 4^m, s = 1{\cdot}5^m, \quad f = \frac{1}{4}$.												
R (m)	σ (mm)	$R\sigma$ (m)	α	$n_1 x_1$ (m)	$n_2 x_2$ (m)	a	b	A_1	A_2	A_3	A_4	A
180	37·5	6·75	32°41′	5	7	−·15	2·175	10·28	·015	·269	2·390	12·95
300	27·5	8·25	32°18′	4·25	10	−·265	2·275	10·25	·02	0	2·275	12·55
600	20	12	31°20′	3·654	10	1·80	1·90	9·70	·42	0	1·90	20·02
1000	17·5	17·5	28°55′	3·684	10	3·25	1·35	9·10	1·223	0	1·35	11·67

Next let the values of A be plotted as ordinates above the corresponding values of $R\sigma$ as abscissœ, and a curve be drawn through the points thus derived; we then obtain the curve, convex downwards, represented in **Fig. 25**, and expressed by the Eqn. $A = \phi(R\sigma)$. In what follows this curve is replaced by the straight line drawn through the end-points. The greatest distance of this straight line from the curve occurs at $R = 600^m$, [*i.e.*, at its middle,] and this distance amounts to ·28 or $\frac{{\cdot}28}{12{\cdot}02} \cdot 100\%$, or about 2·2% of A.

Consequently we have

$$A = 13{\cdot}75 - \frac{1{\cdot}28}{10{\cdot}75} R\sigma$$

and therefore,

$$W = \frac{{\cdot}85938}{R} - \frac{\sigma}{134} \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (42)$$

It is to be noted that in deducing this formula the tension in the couplings has not been taken into account; and for this reason, strictly speaking, it is only valid for *the hind-end of the train.* The inclusion of the effect of the coupling tension in the expression would, however, cause no difficulty in the determination of the coefficient of resistance. The influence of the coupling would tend to reduce resistance; and, consequently, W decreases with the distance of the vehicle from the end of the train, and particularly with increased traction in the couplings. The same train will, accordingly, show a *smaller* resistance on an up-grade than on the level, or even on a falling grade.

The following Table shows in cols. 3 and 4 the curve-resistance for 1 tonne of train-weight calculated from the Formula 36 and 42 for the values of R and σ given in cols. 1 and 2.

1	2	3	4	5
R m	σ $m\,m$	W per 1000 $kg.$ from Eqn. 36	W per 1000 $kg.$ from Eqn. 42	$W = \frac{\cdot 6504}{R - 55} kg.$ per 1000 $kg.$ (von Röckl's Formula.)
180	37·5	4·39	4·49	5·20
200	37·5	4·17	4·02	4·49 (observed 5·00)
300	27·5	2·72	2·65	2·65 (do 2·59)
400	25	1·97	1·96	1·89 (do 1·95)
600	20	1·25	1·28	1·19
800	17·5	·88	·84	·87
1000	17·5	(·65)*	·73	69 (observed, ·69)

This Table shows that the results obtained by the approximate expression (36) agree well with the more exact one (42). We can now conclude, *a postiori*, that also Eqn. 35—from which by the substitution of $d = 4^m$, and the corresponding values of a_1, b_1 and σ, Eqn. 36 is obtained—gives trustworthy results applicable to passenger vehicles of 5^m wheel-base.

And further, that the above-given resistance-coefficients for $d = 5^m$, and the increase of curve-resistance determined from these which would occur if a gross load of 20 tonnes were put on a wheel-base of 5^m instead of on one of 4^m, are correct.

The formula of von Röckl is based on a very large number of experiments,† and gives, as shown in the Table, in curves of greater R than 200^m, almost exactly the same results as our Formula 42. The larger coefficients in col. 5 for curves of less than 300^m R are to be explained by the circumstance that the wheel-tires of the vehicles employed in the Bavarian [or von Röckl] Experiments were much worn, and in sharp curves for this reason a normal working-contact between the leading fore-wheel tread and the head of the outer-rail, such as is postulated in Formula 42, was not possible. If the angle of approach α_1—see § 11—is so large that it does not permit the wheel-tread to rest on the rail at an angle α, then with a given form of wheel-flange, the pressure Y, accompanied by a sliding backwards of the flange on the rail, must act on a contact-surface of which the inclination to the horizontal is greater than α the smaller R is. The curve-resistance becomes accordingly increased.

This increase of W is affected also, of course, by the shape of the head, and will be gone into more minutely in § 22.

The mode of carrying out the Bavarian Experiments, cited here merely for the sake of comparison, corresponds generally with the fundamental assumptions on which Formula 42 is based. In the experiments the vehicles employed were neither quite new nor, on the other hand, were they much worn. The experimental tracks were carefully cleaned-up before each experiment, and they had been run over for so long a time that the rail-heads were quite smooth and polished. The experimental trains, composed of a few vehicles, were shoved-off by locomotives into the experimental horizontal tracks, so that there was no possibility of the occurrence of a coupling tension in any way influential within the meaning of the foregoing discussion.

* The Formula 36 only holds up to $R\,\sigma = 15$.

† Detailed information regarding these Experiments, which were carried out in 1877-78, will be found in the "Zeitschrift für Baukunde," 1880, p. 541.— "Die Versuche der bayerischen Staatseisenbahn über die Widerstände der Eisenbahnfahrzenge bei ihrer Bewegeung in den Gleisen," and an abstract thereof in the "Organ für die Fortschritte des Eisenbahnwesens," 1881, p. 261, etc.

The experimental vehicles in the Bavarian Experiments had the wheel-bases shown in the following Table:

No.	Vehicle.	Wheel-base.	Weight of vehicle.
7	Bavarian Pass. cars, II & III Class...	$3{\cdot}6^m$ to $4{\cdot}4^m$	8·5 to 9· tonnes
4	Bavarian covered waggons ...	$3{\cdot}7^m$	7·0 to 7·7 tonnes
7	Bavarian open goods waggons ...	$3{\cdot}6^m$ to $3{\cdot}7^m$	6·6 to 7· tonnes
2	Hannoverian 6-wheeled open waggons.	$2 \times 3{\cdot}1^m$	7·2 to 7·4 tonnes

Of these wheel-bases some are $\cdot 3^m$ to $\cdot 4^m$ smaller, others ·4 or $2{\cdot}2^m$ greater than the wheel-base of 4^m of Formula 42. What the rôle may be of the various wheel-bases in the results of the von Röckl experiments is not determinable from the above data. But it may certainly be concluded from the above analysis that the von Röckl Formula is not applicable to vehicles of 5^m wheel-base.*

The preceding discussion has been based on the assumption that the vehicle-axles are perfectly parallel to each other: this, however, is never the case in practice.

The subject might be further pursued, and the influence examined into of the possible rotational movement, due to the lateral clearance in the axle-boxes, of the axles relatively to the position of the vehicle assumed in this discussion. However, when the vehicles are maintained in good condition this is of equally small account with the increase of the friction at the axle-journals, and consequently need not be further considered.

§ 20.

Influence of the General Shape of Wheel-tire on Curve-resistance.

The path slid over on the rails by the wheel-pairs depends on the difference $R - \rho$; and this quantity varies with n_1, x_1, n_2, and x_2. A variation in these quantities thus brings about a change in W; and their influence is determined from the following considerations in which d and σ are assumed constant.

*A very full collection of the various formulæ for curve-resistance and of the results of earlier experiments, accompanied by a comparison of the same, is contained in a paper: "Des Longueurs virtuelles d'un tracé de chemin de fer," par M. Charles Baum, Ingénieur des ponts et chaussées: in the "Annales des ponts et chaussées:" 1880, p. 455.

This highly interesting article has supplied the following details:—

(1) Experiments on the Semmering Railway with long and short and trains *of the same weight* yielded the following results.

R *m*	Train of 26 waggons. weighing 198·7 tonnes. W. *kg.*	Train of 13 vehicles, weighing 198·5 tonnes. W. *kg.*
189	3·58	3·03
265	2·57	2·32
284	2·57	2·22

Consequently, curve-resistance *increases* when the number of vehicles is doubled, the total weight moved remaining the same.

(2) Experiments by M. C. Polonceau on the Orléans Railway yielded the following results:—

R *m*	W *kg.*
300	3·90
400	3·30
500	2·75
1000	·75

These experiments were continued by Forquenot with the following results:

R *m*	W *kg.*
300	3·90
500	1·40
1000	0·32

From the equation $a = \frac{R\,\sigma}{n_1 x_1} - s$—see p. 34—it follows that a decreases as $n_1 x_1$ increases; *i.e.*, a diminishes if, with constant x_1, the conicity of the treads and the radius of the rounding-off of rail-top-corner diminishes. If, for example, the tire has not the shape upon which the Table in illustration of Eqn. 41—p. 35—is based, but some other, such that when $R = 600^m$ and $\sigma = 20^{mm}$, $n_1 x_1$ is not equal to 3·654, but to 9·71, then a will be equal to $-$ ·265, instead of 1·80; and A_1 and A_2 will assume the values given in the Table for $R = 300^m$ and $\sigma = 27{\cdot}5^{mm}$ if along with this change $n_2 x_2$ remains = 10. The total resistance A thereby changes from 12·02 to 12·17, or becomes $1\frac{1}{4}$ % greater, and consequently, also, the curve-resistance does so.

If now along with the increase of $n_1 x_1$ to 9·71, $n_2 x_2$ also receives a greater value say, 20, then $A_4 = 1{\cdot}70$, instead of 1·90, and therefore $A = (12{\cdot}17 - {\cdot}2) = 11{\cdot}97$; under these circumstances the resistance then diminishes by $\frac{12{\cdot}02 - 11{\cdot}97}{12{\cdot}02} \times 100\% = {\cdot}4\ \%$.

If, *vice versâ*, $n_1 x_1$ and $n_2 x_2$ have smaller values than those given in the above Table for $R = 600^m$, say $n_1 x_1 = 2{\cdot}57$, for which $a = 3{\cdot}25$, then $A_1 + A_2 = (9{\cdot}10 + 1{\cdot}223) = 10{\cdot}323$; and if $n_2 x_2$ remains 10, then $A = (10{\cdot}323 + 1{\cdot}90) = 12{\cdot}223$ — that is, compared with the values in the Table, there is an increase of $\frac{12{\cdot}223 - 12{\cdot}02}{12{\cdot}02} \times 100\ \% = 1\frac{2}{3}\ \%$ If, also, $n_2 x_2$ is smaller, say = 7, than $A_4 = 2{\cdot}07$, and $A = (10{\cdot}323 + 2{\cdot}07) = 12{\cdot}303$, or there results an increase of 3 %.

It appears, therefore, that in the curve of 600^m R the shape of the wheel-tread does not exercise any important influence on the curve-resistance—always supposing that the flange-hollow is so flatly curved as to permit a due mounting of the Fore-wheel on the head of the rail. These numerical examples show further that in a curve of 600^m R, the curve-resistance is decreased by decreasing of the *inclination of the working surfaces* and the *radius of the flange-hollow;* and that it is increased through the increase of these elements. The same, or the reverse, may occur in another curve according as we start from this or that value of $R\sigma$. It results generally from Eqns. 39 and 40 that for every curve, *i.e.* for every value of $R\sigma$, definite values of contact-angle and curvature of flange-hollow exist for which the curve-resistance of the particular wheel-base is a minimum; and consequently, that an increase of the inclination of the contact-surfaces and of the radius of the flange-hollow by no means necessarily, as might be imagined, results in a reduction of curve-resistance.*

But the investigation of such minimal values is without practical interest, owing to the small influence which any variation, within the ordinary limits, in $n_1 x_1$ and $n_2 x_2$, exercises on curve resistance.

---

* That the amount of the curve-resistance, and consequent proportional wear of tread is very slightly affected by an increase in the flange-hollow radius is shown by an experiment recently carried out by the illustrious Eisenbahn-Director, A Wöhler :—

"Three Coal waggons were procured at one and the same time, all the parts of which came from the same "manufactory. The wheels of two of them were turned-up to experimental profiles, and those of the third to the "ordinary Prussian Standard section. These waggons were so arranged in service that they always ran together with "equal loads in the same train. In this way they travelled some 25,000 *km.* of which some 15,000 *km.* were done on the "Courcelles-Wadgassen Section, on which there are numerous curves of small radius. The radius of the rail top-corner "was 13^{mm}."

The flange-hollow of the two experimental profiles had a hollow of decidedly larger radius than the Prussian Normal profiles. (Fig. 17.)

The results of the experiment were published in the "Centralblatt der Bauverwaltung", 1884, page 179, and the profiles are there shown of the three tires before and after wear. In these profiles there is not any noteworthy difference in the size of the worn cross-sections; and, in other ways, these experiments prove the truth of the remarks in the text on the influence of form of wheel-head.

As to proposals for a new profile with stiffer inclination of the working-surface, and flatter or less rounded flange-hollow, consult the articles of A. Wöhler in the "Zeitschrift für Bauwesen", 1859, page 359, and the "Centralblatt der Bauverwaltung," 1881, page 181—1887, page 177; also the reply to the latter in the (London) "Engineer", 1883, page 157.

§ 21.

The Influence of Wheel-radius on Curve-resistance.

Precisely the same result is reached if the values of $n_1 \mathfrak{r}_1$ and $n_2 \mathfrak{r}_2$ are made 9·71 and 2·52, and 20 and 7, respectively, retaining the values of n_1 and n_2 which form the basis of the Table on p. 35, and giving the radii $\mathfrak{r}_1$ and $\mathfrak{r}_2$ other values.

From the preceding discussion it is evident that a variation in wheel-radius only slightly influences curve-resistance.

What has been determined in the case of a curve of $\mathfrak{R} = 600^m$ can be easily proved to hold good for other curves; as indeed may be seen from the fact that variations in the values of a and b from ·15 to 3·25, and 2·175 to 1·35, respectively, only affect the value of A by 1·28, or about 10 %.

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§ 22.

Influence of the Shape of Flange-Hollow and Rail-head on Curve-resistance.

The determination of the resistance at the leading F-wheel was based on the postulate that the total weight of the wheel rested on a part of the flange-hollow inclined to the horizontal at an angle α. This condition is in a variety of cases not fulfilled—as, for example, when a vehicle which ordinarily runs in the straight, or in a very flat curve, runs by exception through a sharp curve. Through the mutual action of wheel and rail in flat curves and straights the flange-hollow is mainly attacked in the contact-surface indicated by the points a_{I} a_{II} a_{III} **Fig. 26**—situate in the neighbourhood of the transition of the flange into the tread; but in sharp curves the flange is also attacked and worn in the part lying below the point a_{III}. Consequently, when the flange, which on flat curves wears along the line a_I a_{II} a_{III} is made to run through a sharp curve, then the contact at a_{III} must lie in a surface of which the inclination more or less exceeds the angle α. The consequence is that only a part Q_2 of the wheel-load P is supported in the hollow, and the horizontal pressure between flange and rail becomes increased by

$$f\,(P - Q_2)\,\frac{\mathfrak{b}}{\sqrt{e^2 + \mathfrak{b}^2}}.$$

Let α_2 represent the inclination of the surface on which Q_2 rests, and Y the horizontal force which would be produced if Q_2 were $= P$, and $\alpha_2 = \alpha$: then for sharp curves, where $\beta = 0$, approx.

$$Q_2 = \Big[(Y + f\,(P - Q_2)\Big]\,\frac{1 + f \tan \alpha_2}{\tan \alpha_2 - f}:$$

and therefore,

$$Q_2 = (Y + fP).\,\frac{1 + f \tan \alpha_2}{\tan \alpha_2\,(1 + f^2)}. \qquad \ldots \qquad \ldots \qquad (43)$$

If the resistance-coefficient of flange-friction with unworn treads be represented by $\mathfrak{A}_1$, and for worn treads with the above described distribution of load, by $\mathfrak{A}_2$, then

$$\mathfrak{A}_1 = \frac{\tan \alpha \tan \alpha_1}{4};$$

and

$$\mathfrak{A}_2 = \frac{Q_2 \tan \alpha_2 \tan \alpha_1}{4\,P};$$

and

$$\therefore \quad \frac{\mathfrak{A}_2}{\mathfrak{A}_1} = \frac{Y + fP.}{P}\,\frac{1 + f \tan \alpha_2}{\tan \alpha (1 + f^2)} \qquad \ldots \qquad \ldots \qquad (44)$$

Now in the above ratio $\frac{\mathfrak{A}_2}{\mathfrak{A}_1}$, the A_1 of the Table illustrating Eqn. 41a varies: and we obtain for the values of α_2 given in the first column of the following Table, from Eqns. 43, 44, the values therein given of Q_2 and $\frac{\mathfrak{A}_2}{\mathfrak{A}_1}$ for $R = 180^m$, and $d = 4^m$; and further the greater values of A_1—given in 4th column — corresponding thereto.

In addition, column 5 of this Table gives the increase of A_1: column 6, the increase of the curve-resistance in %; and column 7, the curve-resistance for a gross load of 1000 kg. when the angle α_2 increases from 32° 41′ to 60°.

$R = 180^m$. $d = 4^m$						
1	2	3	4	5	6	7
α_2	Q_2	$\frac{\mathfrak{A}_2}{\mathfrak{A}_1}$	A_1	$A_1 - 10{\cdot}28$	$\frac{A_1 - 10{\cdot}28}{12{\cdot}95}$ 100%	W for 1000 kg. kg.
32°41′	P	1·0	10·28	0	0%	4·49
35°	·93 P	1·012	10·41	·13	1%	4·53
40°	·80 P	1·043	10·72	·43	3·4%	4·64
45°	·69 P	1·077	11·07	·79	6·1%	4·76
50°	·60 P	1·119	11·50	1·20	9·4%	4·91
60°	·52 P	1·407	14·47	4·19	32·3%	5·94

The 6th and 7th columns show that if the angle of downward slide increases above the normal size, *i.e., from* $\alpha = 32°41'$, to $\alpha_2 = 45°$, it has but small influence on the magnitude of W. This suggests, therefore, that it might be desirable, as has been proposed by various Engineers, in the case of vehicles of which the leading F-wheel (having an unworn flange-hollow) would run on the round part of the head of the rail at an angle not very different from $\alpha = 32°41'$, to give to the rail and tire a shape in which the slide-angle α_2 should be 45°. With such a shape the area of pressure-surface between flange and rail would be much greater, and thus the very considerable pressure occurring in the present form of wheel-tread and rail-head would be dimiuished.

Such angle of slide of 45° would be quite suitable for locomotive flanges, as the discussion in § 25 shows. On the other hand, the shape shown in Fig. 17a* in which the slide-angle $\alpha_2 = 64°$, must be characterised as a decided mistake; because with this shape W will be increased by more than ⅓rd above the normal amount.

We see from what has preceded that Y and α vary in sharp curves only slightly with R: and what has been shown to hold for $R = 180^m$, holds also for larger radii.

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* As to the usual shape in use on the large English Railways, see the "Engineer," 1883, page 157.

The Providence and Worcester Railroad employs cylindric tires which for half their width are coned, the rounding or hollow of the oblique flange has a radius of $19{\cdot}4^{mm}$ and a rounding-off of the top-corner of rail-head to a radius of $12{\cdot}5^{mm}$. "Railroad Gazette" : 1886, page 180.

§ 23.

The Influence of Flange-clearance on resistance.

With the object of showing how the curve-resistance varies when σ assumes other values, the values of A for curves of 300^m and 600^m for the max. and min. permissible values of σ, are given in the following Table:—

$d = 4^m$; $s = 1{\cdot}5^m$; $f = \frac{1}{4}$.												
R	σ	$R\sigma$	α	$n_1\, x_1$	$n_2\, x_2$	a	b	A_1	A_2	A_3	A_4	A
m	m	m		m	m	m	m					
300	35	10·5	32°18′	5	10	·6	2·05	10·26	·045	0	2·05	12·36
300	20	6	32°18′	3·5	7	·21	2·36	10·35	·057	·568	2·173	13·15
600	30	18	31°20′	4·61	10	2·40	1·30	9·40	·690	0	1·30	11·39
600	15	9	31°20′	3	10	1·5	2·20	9·86	·277	0	2·20	12·34

This Table shows that in curves of 300^m rad. when σ diminishes from $35^m/_m$, to $20^m/_m$, A increases from 12·36 to 13·15; that is, by $\frac{13{\cdot}15 - 12{\cdot}36}{12{\cdot}55}\,100\% = 6\%$, of its value for the mean clearance. For curves of 600^m rad. when σ diminishes from $30^m/_m$ to $15^m/_m$, this increase amounts to $\frac{12{\cdot}34 - 11{\cdot}39}{12{\cdot}02} \times 100\,\% = 8\,\%$ of the mean resistance.

If Eqn. 42 be employed to examine the influence of σ on curve-resistance, then, as compared with the preceding results, we obtain for $R = 300^m$ and $\sigma = 20^m/_m$ a greater value of W by 4·2 % than for $R = 300^m$ and $\sigma = 35^m/_m$: and, further, for $R = 600^m$ and $\sigma = 15^m/_m$., a larger value by 8·8 % than for $R = 600^m$ and $\sigma = 30^m/_m$. From this it is evident that the influence of σ on the curve-resistance is correctly expressed by the Formula 42 for vehicles of 4^m wheel-base, and that this formula is valid for *any wheel-base within the usual limits of clearance.*

In contradiction with our statement of the influence of σ as above determined by calculation is the statement on this point in the above-cited account of the Bavarian Experiments: viz. that when only half the amount of the usual curve track-widening is employed there is almost exactly the same resistance as when there is none; and, that when the usual normal curve-enlargement for a curve of from 300^m to 550^m R is given there is a diminution of resistance amounting on an average to 25 %.

It is clear from the evidence afforded by the above numerical determinations that such a very considerable increase in curve-resistance as a result of a diminution of the clearance σ, observed by von Röckl, can only have been due to special and quite accidental causes which, presumably, might have been avoided by a more careful running in the contracted gauge.

§ 24.

The magnitude of Curve-resistance at the Flange of the Fore-wheel.

In the Table constructed from Eqn. 41*a*:

A — represents the total resistance:
A_1 — the curve-resistance of the leading F-wheel: } in the flange-hollow
A_3 — the resistance of the inner (guiding) H-wheel: }
A_2 — that of the F. A. } at the contact-surface of their treads.
A_4 — that of the H. A. }

Expressing the values of the above symbols given in the Table as % of the whole resistance A, we obtain the figures exhibited in the following Table.

R m	A_1 as % of A	$A_2+A_3+A^4$ as % of A
180	79	21
300	81	19
600	80	20
1000	77	23

Whence we see that *about 80 % of the total vehicle resistance occurs at the leading F-wheel flange* and the outer-rail. Accordingly, it is at this point that the wear of rail in curves reaches its maximum, viz., at the inner-side of the outer-rail.

On many Railways lubrication by *greasing* the flanges of locomotives has been employed to reduce the wear at the leading flange. The greased tires acquire with time a polished flange-hollow, and thus the abrasion is got rid of which without lubrication would take place in the flange-hollow (in curves) on the rounded top-corner of rail-head.

The life of the wheel-tread on a road having many curves is prolonged by greasing by some 40 to 50 %; and on the surface of the rails thus smoothed by the greased locomotive wheel-treads the fore-wheels of succeeding vehicles slide more easily; thus causing a reduction of curve-resistance in vehicles also.

No alteration in the locomotive tractive-force arising from the greasing of the flange-hollow has ever been observed.

The greasing of locomotive flanges has been introduced on the Austrian State Railway Company, the Kaiserin Elisabeth Railway, the Austrian North-West Railway, the Bavarian State Railway, the Upper Hessian Railway, and the South Austrian Railway, and others.*

The locomotives of the Berlin City and Suburban Railway are provided with an injector arrangement by means of which the flange-hollows can, when required, be wetted with *water*.

NOTE.—Particular attention was devoted to observing the effects of the greasing of the inner-side of the rail-head of the outer-rail in the above-cited Bavarian Experiments. The experiments showed that by greasing, the curve-resistance was reduced on curves of $R = 300^m$ to 550^m, on an average by 49 %; in curves of 200^m to 150^m R, by 54 %, and in a curve of 100^m by 61 %.

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* "Organ für die Fortschritte des Eisenbahnwesens": 1879 p. 106: 1878, p. 3: and "Zeitschrift des oesterr. Ingenieur- und Architekten- Vereins:" 1878, p. 194.

Fig. 27.

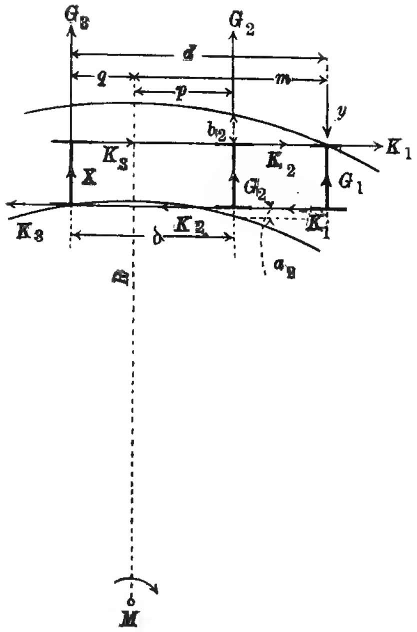


Fig. 28.

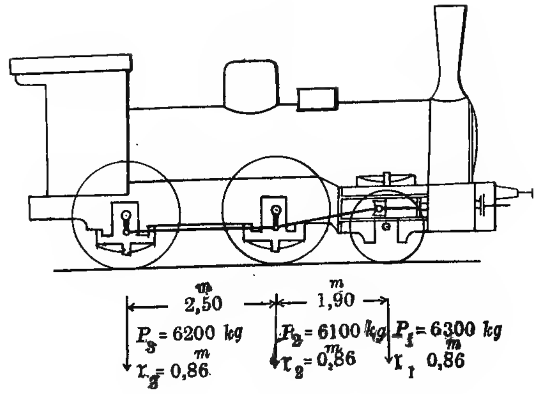


Fig. 29.

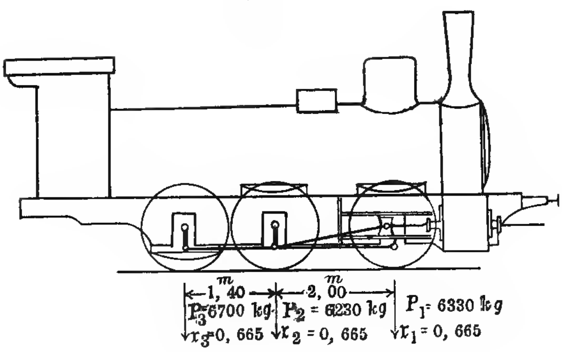
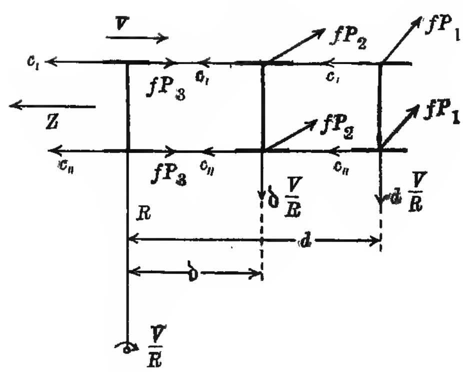


Fig. 30.



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CHAPTER III.

3-AXLED LOCOMOTIVES IN CURVES.

§ 25.

The Action of Locomotives on the Track: Influence of Motion in a Curve on the Tractive Force.

When determining the forces which during the travel of a locomotive in a curve act between wheel and rail in the direction of the vehicle's motion and at right-angles thereto (*i.e.*, parallel to the axle), the tractive action of the locomotive must be taken into account, because the tractive force affects the forces K, G, Y, and X.

Consequently, these forces must be separately determined, according as the locomotive is light-running, or is hauling a train.

(*a*) The light-running Locomotive.

For the undisplaceable M. A. of a 3-axled vehicle—referring to **Fig. 27**—we have

$$b_2 = \frac{m^2}{2R} - \frac{p^2}{2R}:$$

and referring to Fig. 13,

$$r - r_1 = \frac{a_2 - b_2}{n_2} = \frac{\sigma - \dfrac{m^2 - p^2}{R}}{n_2}:$$

and so—from Eqn. 12*b*—

$$c_2 = \frac{V}{2}\left[\frac{\sigma - \dfrac{m^2 - p^2}{R}}{n_2\, x_2} - \frac{s}{R}\right]$$

or, if $m^2 - p^2$ is put $= \lambda^2$,

$$c_2 = \frac{V}{2R}\cdot\left(\frac{R\sigma - \lambda^2}{n_2\, x_2} - s\right).$$

Further, we have

$$p = \mathfrak{d} - q, \quad \text{and — Eqn. 17,}\ q = \frac{d}{2} - \frac{R\sigma}{d}.$$

Now if for the purposes of the following discussion in finding the value of Y we make the inexact, but still permissible, assumption that the H. A. of the locomotive stands radially when $R\,\sigma > \frac{d^2}{2}$, then for the above locomotive q becomes $= 0$ when $R\,\sigma$ has the values given in the following Table.

Vehicle.	d m	$q = o$, when $R\sigma =$
Pass. Locomotive, Prussian S. Rys.	4·4	9·68
Goods Locomotive, do do	3·4	5·78

From the general expressions, Eqns. 13, 14, we obtain for the M. A.

$$G_2 = f\,P_2 \frac{p}{\sqrt{p^2 + \frac{1}{4}\left(\dfrac{R\,\sigma - \lambda^2}{n_2\, x_2} - s\right)^2}} \quad \cdots \quad \cdots \quad (45)$$

$$K_2 = f\,P_2 \frac{\frac{1}{2}\left(\dfrac{R\,\sigma - \lambda^2}{n_2\, x_2} - s\right)}{\sqrt{p^2 + \frac{1}{4}\left(\dfrac{R\,\sigma - \lambda^2}{n_2\, x_2} - s\right)^2}}. \quad \cdots \quad \cdots \quad (46)$$

From **Fig. 27,** we obtain for Y the relation

$$Y = G_1 - K_1 \frac{s}{d} + 2\,G_2 \frac{\mathfrak{v}}{d} - K_2 \frac{s}{d} - K_3 \frac{s}{d}\,.$$

And if instead of $G_1 - K_1 \frac{s}{d}$, and $K_3 \frac{s}{d}$, the first and second numbers of Eqn. 15—changing the values $P_2\, n_2\, x_2$ into $P_3\, n_3\, x_3$ — are inserted, and for $2\;G_2 \frac{\mathfrak{v}}{d} - K_2 \frac{s}{d}$, the corresponding expression from Eqns. 45, 46 — then we obtain generally,

$$Y = f\,P_1 \frac{m - \frac{s}{2\,d}\left(\frac{R\,\sigma}{n_1\,x_1} - s\right)}{\sqrt{m^2 + \frac{1}{4}\left(\frac{R\,\sigma}{n_1\,x_1} - s\right)^2}} + f\,P_2 \frac{2p\,\frac{\mathfrak{v}}{d} - \frac{s}{2\,d}\left(\frac{R\,\sigma - \lambda^2}{n_2\,x_2} - s\right)}{\sqrt{p^2 + \frac{1}{4}\left(\frac{R\,\sigma - \lambda^2}{n_2\,x_2} - s\right)^2}} +$$

$$+ fP_3 \frac{\frac{s}{2\,d}\left(\frac{R\sigma}{n_3\,x_3} - s\right)}{\sqrt{q^2 + \frac{1}{4}\left(\frac{R\sigma}{n_3\,x_3} - s\right)^2}} \quad \ldots \quad \ldots \quad (47)$$

For the case in which $R\,\sigma \gtreqless \frac{d^2}{2}$, the last term of this expression becomes $\pm f\,P_3 \frac{s}{d}$; and the minus sign holds if the apex of the rolling-cone of the H. A. lies between the track and its centre; and the plus sign, for any other position of this rolling-cone apex.

According to Eqn. 18, this term is therefore positive when $R\sigma < d^2 + n_3 x_3 s$; and negative if $R\,\sigma > d^2 + n_3\,x_3\,s$. Further, it is zero when $R\sigma = d^2 + n_3\,x_3\,s$. This occurs for the forwards-running Passenger Locomotive, when

$$R\sigma = 4{\cdot}4^2 + 20 \times {\cdot}86 \times 1{\cdot}5 = 45{\cdot}16:$$

and with the forwards-running Goods Engine, when

$$R\sigma = 3.4^2 + 20 \times {\cdot}66 \times 1{\cdot}5 = 31{\cdot}36.$$

Now, we have—from Eqn. 47, and with the assistance of Eqns. 19, 20, and proceeding in the same way as in the determination of Y for 2-axled vehicles—for the forwards-running Passenger Engine, shown in **Fig. 28,** when

$x_1 = {\cdot}56^m$; $x_2 = x_3 = {\cdot}86^m$; $P_1 = 6300$ *kg*; $P_2 = 6100$ *kg*; $P_3 = 6200$ *kg*; $d = 4{\cdot}4^m$; and $\mathfrak{v} = 2.5^m$—the following values for Y and α:

$R = 180^m$ $R\sigma = 6{\cdot}75^m$ $\sigma = {\cdot}0375^m$.	$Y = {\cdot}99\,fP_1$ $+\;1{\cdot}17\,fP_2$ $+\;{\cdot}29\,fP_3$ $\therefore\; Y = 2{\cdot}41\,fP_1$	$\alpha = 45°$.
$R = 2580^m$ $R\sigma = 45{\cdot}16^m$ $\sigma = {\cdot}0175^m$.	$Y = {\cdot}447\,f\,P_1$ $+\;1{\cdot}064\,fP_2$ $+\;{\cdot}343\,fP_3$ $\therefore\; Y = 1{\cdot}82\,fP_1$	$\alpha = 28°12'$

In evaluating the above, the flange-hollow radius—somewhat sharp-worn—is assumed to be as before $12^m/_m$, and the conicity of tread $= \frac{1}{20}$.

From the above-found two values, and on the assumption (of which the accuracy has been proved by numerical trial) that as R increases from $= 180^m$ to $R = 2580^m$, Y correspondingly diminishes from $2{\cdot}41\,fP_1$ to $1{\cdot}82\,fP_1$ we obtain the formula

$$Y = \left(2{\cdot}454 - \frac{R}{4070}\right) f\,P_1 \quad \ldots \quad \ldots \quad \ldots \quad (48)$$

In the same way, and with the same assumptions, we obtain for the forwards-running Goods Engine of **Fig. 29** :—

$$R = 180^m \qquad Y = 1{\cdot}0 f_1 P_1 \qquad \alpha = 45^\circ$$
$$R\sigma = 6{\cdot}75^m \qquad + 0{\cdot}939 f P_2$$
$$\sigma = {\cdot}0375^m. \qquad + 0{\cdot}441 f P_3$$
$$\therefore\ Y = 2{\cdot}39 f P_1$$

$$R = 1796^m \qquad Y = {\cdot}476 f P_1 \qquad \alpha = 31^\circ 20'$$
$$R\sigma = 31{\cdot}36^m \qquad + {\cdot}760 f P_2$$
$$\sigma = {\cdot}0175^m. \qquad + {\cdot}441 f P_3$$
$$\therefore\ Y = 1{\cdot}69 f P_1$$

And the above maximum value of $R\sigma$, viz. $R\sigma = 31{\cdot}36$, is the corresponding value of Eqn 18. From the above two values of Y we obtain

$$Y = \left(2{\cdot}47 - \frac{R}{2310}\right) f P_1 \qquad \dots \qquad \dots \qquad \dots \qquad (49)$$

From the Eqns. 48, 49, 19 and 20, we get the values of Y and α exhibited in the following Table.

Vehicle.	$f = \frac{1}{4}$.					
	$R = 180^m$ $R\sigma = 6{\cdot}75$		$R = 300^m$ $R\sigma = 9{\cdot}0$		$R = 600^m$ $R\sigma = 12{\cdot}0$	
	Y	α	Y	α	Y	α
Passenger Loco: running forwards ...	$2{\cdot}41 f P_1$	45°	$2{\cdot}38 f P_1$	44°48′	$2{\cdot}30 f P_1$	43°43′
Goods Loco: running forwards ...	$2{\cdot}39 f P_1$	45°	$2{\cdot}34 f P_1$	44°48′	$2{\cdot}21 f P_1$	42°50′

Further, we obtain from the Eqns. 26 and 45 the following values for the force G_1 tending to cant the inner-rail at the inner Fore-wheel, and for G_2 at the inner Middle wheel,—

Vehicle.	$R = 180^m$ $R\sigma = 6{\cdot}75$		$R = 300^m$ $R\sigma = 9{\cdot}0$		$R = 1000^m$ $R\sigma = 17{\cdot}50$		$R = 1500^m$ $R\sigma = 26{\cdot}25$	
	G_1	G_2	G_1	G_2	G_1	G_2	G_1	G_2
Pass. Loco: running forwards ...	$1 f P_1$	${\cdot}92 f P_2$	$1 f P_1$	${\cdot}94 f P_2$	${\cdot}88 f P_1$	${\cdot}97 f P_2$	${\cdot}78 f P_1$	${\cdot}99 f P_2$
Goods Loco: running forwards ...	$1 f P_1$	${\cdot}84 f P_2$	$1 f P_1$	${\cdot}88 f P_2$	${\cdot}83 f P_1$	${\cdot}94 f P_2$	${\cdot}79 f P_1$	${\cdot}98 f P_2$

As regards the horizontal forces acting at the Hind-wheels, tending to cant the rails, we must distinguish whether $R\sigma$ is $\overset{<}{\underset{>}{=}} \frac{d^2}{2}$. When $R\sigma < \frac{d^2}{2}$, we have for determining X the relations

$$X = Y - (G_1 + 2\,G_2 + G_3) \qquad \dots \qquad \dots \qquad (50)$$

and

$$G_3 = -\frac{q}{\sqrt{q^2 + \frac{1}{4}\left(\frac{R\sigma}{n_3\,x_3} + s\right)^2}}\, f P_3 \qquad \dots \qquad (51)$$

When $R\sigma \geqq \frac{d^2}{2}$, then $X = G_3$

and

$$G_3 = -\frac{G_1 + 2G_3 - Y}{2} \quad \ldots \quad \ldots \quad \ldots \quad (52)$$

We accordingly obtain the following Table for X and G_3.

Vehicle.	$R = 180^m$ $R\sigma = 6{\cdot}75$	300^m 9	1000^m $17{\cdot}50$	1500^m $26{\cdot}25$
Passenger Loco: running forwards ...	$X = {\cdot}13 f P_3$	$-{\cdot}20 f P_3$	$-{\cdot}35 f P_3$	$-{\cdot}38 f P_3$
$\left[\frac{d^2}{2} = 9{\cdot}68\right]$ ...	$G_3 = -{\cdot}5 f P_3$	$-{\cdot}20 f P_3$	$-{\cdot}35 f P_3$	$-{\cdot}38 f P_3$
Goods Loco: running forwards ...	$X = -{\cdot}15 f P_3$	$-{\cdot}18 f P_3$	$-{\cdot}34 f P_3$	$-{\cdot}47 f P_3$
$\left[\frac{d^2}{2} = 5{\cdot}78\right]$ ...	$G_3 = -15 f P_3$	$-{\cdot}18 f P_3$	$-{\cdot}34 f P_3$	$-{\cdot}47 f P_3$

With the aid of the values of G_3 given above we can determine what amount of error is involved in the assumption that the H. A. stands radial when $R\sigma > \frac{d^2}{2}$.

On this assumption we have $K_3 = f P_3$: whereas from the preceding, for $R = 300^m$,

$$K_3 = f P_3 \sqrt{1 - {\cdot}2^2} = f P_3 \, {\cdot}979:$$

and for $R = 1000^m$,

$$K_3 = f P_3 \sqrt{1 - {\cdot}35^2} = f P_3 \, {\cdot}936.$$

Consequently, the more accurate values of K_3 differ very little from $f P_3$. The error in values of Y — calculated on the assumption that $K_3 = f P_3$ — is therefore insignificant, and may be neglected.

The action above determined of the locomotive on the rails occurs when the forces between wheel and rail are in equilibrium—viz., when running without load, or in special circumstances, as when running down-hill; the engine's tractive force in the draw-bar at the head of the train is then zero.

(b) Locomotive hauling a Train.

Suppose the engine to exert a tractive force Z in the draw-bar. Then for the forces between rail and wheel acting longitudinally, instead of $\Sigma K = 0$, we have $\Sigma K = Z$. Consequently, K_1, K_2, K_3, have other values than when the locomotive runs without load. Now as G_1, G_2, and G_3 vary with K_1, K_2, and K_3, they, also, must have altered values, Consequently, Y also must undergo a change in amount.

The maximum tractive force Z depends on the power of the engine and on the magnitude and direction of the frictional resistance between the driving-wheels and the rails.

If the steaming power of the engine be so great that in any case the whole adhesion can be fully utilized as T. F., then the maximum T. F. depends only on the load on the driving-wheels, the coefficient of friction, and on the direction of the unavoidable sliding of the driving-wheels on the rails which accompanies motion in a curve.

Under the circumstances stated, the maximnm Tractive Force, Z, is determined by the nature of the locomotive's motion.

In order to illustrate these relations, we shall consider in detail the motion of a 3-coupled Goods-locomotive in its travel through a curve. And to simplify matters we shall assume cylindric treads.

Let the outer wheels slide with the velocity c_1 on the rails in the direction shown in **Fig. 30**; then the inner wheels slide with a velocity

$$c_{11} = c_1 + s\frac{V}{R}$$

and

$$Z = fP_3(1+\epsilon) + fP_2\left[\frac{c_1}{\sqrt{c_1^2 + \left(b\frac{V}{R}\right)^2}} + \frac{c_1 + s\frac{V}{R}}{\sqrt{\left(c_1 + s\frac{V}{R}\right)^2 + \left(b\frac{V}{R}\right)^2}}\right] +$$

$$+ fP_1\left[\frac{c_1}{\sqrt{c_1^2 + \left(d\frac{V}{R}\right)^2}} + \frac{c_1 + s\frac{V}{R}}{\sqrt{\left(c_1 + s\frac{V}{R}\right)^2 + \left(d\frac{V}{R}\right)^2}}\right] \quad \ldots (53)$$

In the above expression, ϵ represents a coefficient for the outer H-wheel, which has the following values :—

$$\epsilon = -1, \text{ if } c_1 < o;$$
$$\epsilon = 0, \text{ if } c_1 = o;$$
$$\epsilon = +1, \text{ if } c_1 > o.$$

From the above equation we have

	H. A.	M. A.	F. A.	
for $c_1 = -\frac{s}{2}\frac{V}{R}$:	$Z = o.$			
$c_1 = 0$:	$Z = fP_3$	$+ \cdot 73\, fP_2$	$+ 0\cdot 40\, fP_1$	$= \cdot 36\, fQ.$
$c_1 = s\frac{V}{R}$:	$Z = 2fP_3$	$+ 1\cdot 64\, fP_2$	$+ 1\cdot 04\, fP_1$	$= \cdot 78\, fQ.$
$c_1 = 2s\frac{V}{R}$:	$Z = 2fP_3$	$+ 1\cdot 86\, fP_2$	$+ 1\cdot 46\, fP_1$	$= \cdot 89\, fQ.$
$c_1 = 4s\frac{V}{R}$:	$Z = 2fP_3$	$+ 1\cdot 95\, fP_2$	$+ 1\cdot 62\, fP_1$	$= \cdot 93\, fQ.$

wherein $Q = 2\,(P_1 + P_2 + P_3) = 2\,(6300 + 6200 + 6700) = 38400$ kg. = the total adhesion-weight.

The values of c_1 and of Z the tractive force are represented graphically in **Fig. 31**, from which it is seen that an increase in c_1, say something over $2\,s\frac{V}{R}$, has a very insignificant effect on Z.

NOTE.—With the help of this diagram we can determine, conversely, what velocity c_1 occurs in a curve when the locomotive is exerting a given T. F., and also what the influence of the coef. of friction is thereon.

Suppose, for example, $Z = \frac{Q}{8}$ when $f = \frac{1}{4}$. Then by the vertical scale of Fig. 31, $Q = 4 \times 2 = 8^{cm}$; and therefore $Z = \frac{Q}{8} = 1^{cm}$. Now for $Z = 1^{cm}$ we find in the Fig. $c_1 = \cdot 3s\frac{V}{R}$; and the corresponding values are consequently, $Z = \frac{Q}{8}$, $f = \frac{1}{4}$, and $c_1 = \cdot 3s\frac{V}{R}$.

In this way, the values of c_1 exhibited in the following Table have been obtained, such that for given values of f, the tractive force $Z = \frac{Q}{8}$.

$f = \frac{1}{4}$	$\frac{1}{5}$	$\frac{1}{6}$	$\frac{1}{7}$	$\frac{1}{8}$
$c_1 = 3s\frac{V}{R}$	$= \cdot 5s\frac{V}{R}$	$= \cdot 9s\frac{V}{R}$	$= 2s\frac{V}{R}$	$= \infty$

If we insert in Eqn. 53

$$c_1 = ns\,\frac{V}{R},$$

then we obtain

$$Z = f\,P_3\,(1+\epsilon) + f\,P_2\left(\frac{ns}{\sqrt{n^2 s^2 + \mathfrak{d}^2}} + \frac{ns+s}{\sqrt{(ns+s)^2 + \mathfrak{d}^2}}\right) + f\,P_1\left(\frac{ns}{\sqrt{n^2 s^2 + d^2}} + \frac{ns+s}{\sqrt{(ns+s)^2 + d^2}}\right).$$

From this equation we see that Z increases with n. Now $n = \frac{c_1}{V}\,\frac{R}{s}$ and is thus greater for the same value of $\frac{c_1}{V}$ the greater R is: and conversely, for the same T. F., $\frac{c_1}{V}$ is smaller the greater R is. Consequently, with the exertion of the same tractive force the driving-wheels of locomotives have smaller paths of longitudinal slide in flat curves than in sharp curves.

The preceding values of Z (p. 47) calculated from $c_1 = -\frac{sV}{2R}$, up to $c_1 = 4s\frac{V}{R}$ show that the individual driving-wheels contribute in different degrees to the production of tractive force: and we see that the F. A.—in consequence of its greater movement in its own direction—contributes the least.

In order to bring this out more clearly, the contributions of M.A. and F.A. to the total tractive force are represented graphically in **Fig. 32** in per cents. of $2\,f\,P_2$, and $2\,f\,P_1$, respectively, corresponding to various velocities c_1.

The resulting curves show, for example, that for $c_1 = 2s\frac{V}{R}$, *i.e.*, when $Z = {\cdot}89\,f\,Q$, or say, 90 % of the whole adhesion, only 73 % of the adhesion at the F. A. is utilizable. Further, they show that if c_1 increase to $4s\,\frac{V}{R}$, and thus Z increases to 93 % of $f\,Q$, the utilization of the adhesion of the F. A. only rises to 80 %. And since a locomotive is but rarely called upon to exert 93 % of its total adhesion, it follows that when moving in curves not more than 80 % of the F. A. adhesion can be utilized.

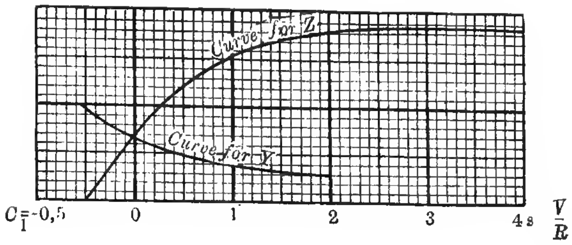
In straights, also, where the (unavoidable) oscillation of the engine horizontally increases with the T. F. the same conditions apply to the F. A. as in curves.

These considerations show that as regards the T. F. to be exerted by a 3-axled engine having 2 coupled-axles, it is not desirable to make the F. A. a driving-axle, and the H. A. a trailing-axle.

If the F. A. is a driving-axle, and a part, A, of the adhesion, $f\,P_1$, of the F-wheel is available for T. F., then there remains the component $\sqrt{f^2\,P_1{}^2 - A^2}$ as resistance to the displacement of the wheel in the direction of the F. A. The resistance opposed by the F. A. to its displacement in its own direction—*i.e.*, transversely to the track—diminishes in the same degree as the F. A. co-operates in the production of the T. F. Consequently, the F. A. resists such a displacement most effectively when it does not co-operate at all in the production of T. F. It will best fulfil its duty as leading axle, *i.e.*, it will most securely guide and maintain the vehicle in its direction of motion and most energetically oppose horizontal oscillations, when it is *not* a driving-axle.

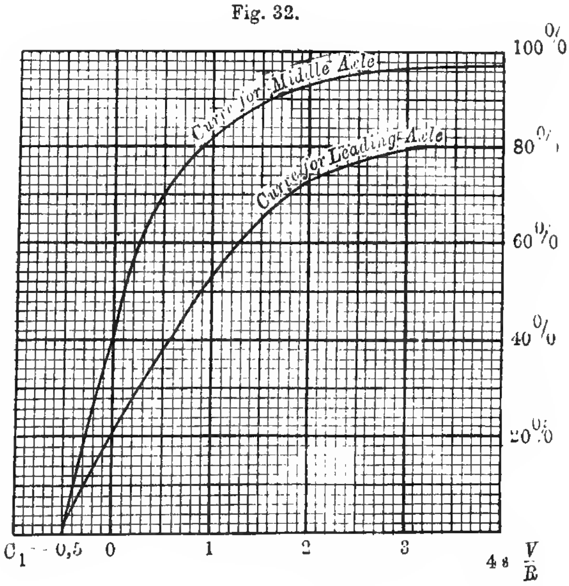
For this reason locomotives having an uncoupled axle in front run more smoothly, other circumstances being equal, than when the fore-axle is a driving-axle, and for this reason Express locomotives always have the fore-axle free.

Fig. 31.



Vertical Scale f Q = 2 cm.

Fig. 32.



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For the pressure Y, we have, on the above assumptions,

$$Yd = Z\frac{s}{2} - s\left\{\epsilon f P_3 + f P_2 \frac{c_1}{\sqrt{c_1^2 + \left(\mathfrak{b}\frac{V}{R}\right)^2}} + f P_1 \frac{c_1}{\sqrt{c_1^2 + \left(d\frac{V}{R}\right)^2}}\right\} +$$

$$+ \mathfrak{b} f P_2 \left\{\frac{\mathfrak{b}\frac{V}{R}}{\sqrt{c_1^2 + \left(\mathfrak{b}\frac{V}{R}\right)^2}} + \frac{\mathfrak{b}\frac{V}{R}}{\sqrt{\left(c_1 + s\frac{V}{R}\right)^2 + \left(\mathfrak{b}\frac{V}{R}\right)^2}}\right\} + d f P_1 \frac{d\frac{V}{R}}{\sqrt{\left(c_1 + s\frac{V}{R}\right)^2 + \left(d\frac{V}{R}\right)^2}} : \quad (54)$$

and, accordingly,

$$\begin{array}{llll} \text{for } c_1 = -\frac{s}{2}\frac{V}{R}: & & Y = 2{\cdot}44\, f P_1 & = {\cdot}50\, f Q. \\ c_1 = 0 & : & Y = 2{\cdot}08\, f P_1 & = {\cdot}34\, f Q. \\ c_1 = s\frac{V}{R} & : & Y = 1{\cdot}29\, f P_1 & = {\cdot}21\, f Q. \\ c_1 = 2s\frac{V}{R} & : & Y = 0{\cdot}92\, f P_1 & = {\cdot}15\, f Q. \end{array}$$

If therefore $\quad c_1 = -\frac{s}{2}\frac{V}{R}$, and thus by Eqn. 53, $Z = 0$,

then $\quad Y = 2{\cdot}44\, f P_1$ in all curves.

On the other hand, with conical tires, $Y = 2{\cdot}39\, f\, P_1$ in a curve of 180^m R; and it is correspondingly less in curves of greater radius.

Whence we see that with conical treads the pressure Y is smaller than with cylindric.

In Fig. 31 the numerical values of Eqn. 54 for a locomotive hauling a load are represented graphically above the corresponding values of c_1 as abscissœ; and it is thus self-evident that Y decreases as Z increases.

In deducing the Eqn. 54, it has been tacitly assumed that the locomotive is advanced *uniformly* by the action of the two cylinders, and that Y in any particular case has a constant value. This is not so in reality, because the driving force of the locomotive is at any instant greater in the driving-wheels of one side than in those of the other; and consequently, a horizontal moment of rotation is produced.

The values of Y obtained by calculation are accordingly simply *mean values*, about which the real value of the horizontal pressure varies. These fluctuations from the mean values are the greater the more powerful the locomotive, *i.e.* the greater the T. F. is.

Now because Y diminishes as Z increases, it may happen that the value of Y is entirely within the fluctuation limits, and at some instant may become zero if the moment of rotation within the body of the engine due to the driving force chances to act in the same sense as the rotation of the engine due to the motion in the curve.

This will be approximately the case if the Y of Eqn. 54 is so small that

$$Yd = Z\frac{s}{2},$$

or

$$Y = Z\frac{s}{2d} = {\cdot}22\, Z\,.$$

We see from Fig. 31 that this happens when $Z = {\cdot}8\, f Q$. With a greater value of Z, the well-known oscillation of the locomotive about a vertical axis arises. A small wheel-base promotes the occurrence of these oscillations.

With the decrease of Y with increased loading of the locomotive is associated the decrease of the angle α of the surface-contact at the leading F-wheel. This angle can thus assume various values in the same curve and in the same locomotive. This fact has an important bearing on the choice of the best form of flange for locomotive wheels, and for the corresponding shape of rail-tread. (See also p. 40).

§ 26.

Curve-resistance of a Goods-Locomotive.

The values of Y and Z obtained from the Eqns. 53, 54, for given values of c_1 may be further made use of to determine the resistance experienced by a Goods-engine, and how far this resistance depends on the T. F. developed, as follows.

By precisely the same method of approximation as was used to determine the curve-resistance of a 4-wheeled vehicle, we obtain the coefficient of the resistance arising from the on-running of the F-wheel against the outer-rail: *viz.*

$$W_1 = \frac{P_1 \tan \alpha \tan \alpha_1}{Q} .$$

Further, owing to the sliding of the 6 driving-wheels in the longitudinal direction, the loss of energy per sec., when $c_1 = -\frac{s}{2}\frac{V}{R}$, is

$$f\,(P_1 + P_2 + P_3)\,\frac{s\,V}{R} :$$

and the corresponding coef. of resistance is

$$W_2 = f\,\frac{P_1 + P_2 + P_3}{Q}\,\frac{s}{R} = \frac{f\,s}{2\,R} .$$

Thus we have

$$\text{for } c_1 = 0, \qquad W_2 = \frac{f\,s}{2\,R};$$

$$c_1 = s\,\frac{V}{R}, \qquad W_2 = \frac{3}{2} f\,\frac{s}{R};$$

$$c_1 = 2\,s\,\frac{V}{R}, \qquad W_2 = \frac{5}{2} f\,\frac{s}{R}.$$

If we substitute in the expression for W_1

$$\frac{P_1}{Q}\tan\alpha_1 = \frac{6300}{38400}\cdot\frac{3{\cdot}4}{R} = \frac{{\cdot}558}{R},$$

then

$$W_1 = \frac{{\cdot}558}{R}\tan\alpha.$$

Further, tan α is given by the expression,—Eqn. 19—

$$\tan\alpha = \frac{\frac{Y}{P_1} + f}{1 - \frac{Y}{P_1} f} = \frac{\frac{Y}{P_1} + \frac{1}{4}}{1 - \frac{Y}{P_1}\frac{1}{4}}$$

calculated for the value of Y corresponding to the given values of c_1 the velocity of slide; and finally, if $s = 1{\cdot}5^m$, and $f = \frac{1}{4}$ are inserted in the expression for W_2 we obtain the values of W_1, W_2, given in the following Table:—

c_1	Z	$f = \frac{1}{4}$		
		W_1	W_2	$W = W_1 + W_2$
$-\frac{s}{2}\frac{V}{R}$	0	$\frac{{\cdot}5662}{R}$	$\frac{{\cdot}1875}{R}$	$\frac{{\cdot}7537}{R}$
0	$\cdot36\,f\,Q$	$\frac{{\cdot}4939}{R}$	$\frac{{\cdot}1875}{R}$	$\frac{{\cdot}6914}{R}$
$s\frac{V}{R}$	$\cdot78\,f\,Q$	$\frac{{\cdot}3480}{R}$	$\frac{{\cdot}5625}{R}$	$\frac{{\cdot}9105}{R}$
$2s\frac{V}{R}$	$\cdot89\,f\,Q$	$\frac{{\cdot}2842}{R}$	$\frac{{\cdot}8375}{R}$	$\frac{1{\cdot}1217}{R}$

The last column of this Table gives the total resistance $W = (W_1 + W_2)$ of the Goods locomotive.

From the expressions for W in that Table we obtain, by division, the following Table of coefficients of resistance for curves of 180^m, 300^m, 600^m, and 800^m R.

c_1	Z	R				REMARKS.
		180^m	300^m	600^m	800^m	
$-\frac{s\,V}{2\,R}$	0	·00419 (168)	·00277 (111)	·00125 (50)	·00094 (38)	The figures in brackets give the curve-resistance of the whole locomotive in *kilogs:* in calculating it, the weight of the engine *without tender* is taken as 40 tonnes.
0	·36 $f\,Q$	·00390 (156)	·00230 (92)	·00115 (46)	·00086 (34)	
$s\frac{V}{R}$	·78 $f\,Q$	·00506 (202)	·00303 (121)	·00152 (61)	·00114 (46)	
$2s\frac{V}{R}$	·89 $f\,Q$	·00624 (250)	·00374 (150)	·00187 (75)	·00140 (56)	

These coefficients show that the curve-resistance of the Goods-engine is affected in a high degree by the T. F. developed: and further, the first of these Tables shows that the smallest coefficients occur for $Z = {\cdot}36\,f\,Q$; and that for $Z = {\cdot}89\,f\,Q$, the curve-resistance is $\frac{1{\cdot}1217}{0{\cdot}6914}$ or 1·6 times as great as for $Z = {\cdot}36\,f\,Q$.

Further, a comparison with the resistance-coefficients of the 2-axled vehicle of 4^m wheel-base shows that the per-cent. curve-resistance of a Goods-locomotive for $Z = 0$ is almost as large as the curve-resistance of the 2-axle 4^m vehicle; but that for $Z = {\cdot}36 f Q$ it is smaller; and for $Z = {\cdot}89\,f\,Q$ is about 40 % greater.

§ 27.

The Variations in Wheel-load during motion in a Curve.

If the diameters of the wheels of a locomotive are not *exactly equal* to one another then the forces acting at their circumferences in the axle-direction produce in the body of the engine a torsion-moment, acting in a vertical plane, which is given by the expression

$$M = 2\,G_2\,(x_2 - x_1) + (X + G_3)\,(x_3 - x_1)$$

Owing to this moment, the loads on the bearing-springs on the curve-centre side of the locomotive will be increased by $\frac{P_1}{n}, \frac{P_2}{n}, \frac{P_3}{n}$, respectively, and the opposite springs will be correspondingly unloaded by the same amounts. From **Fig. 33**, we have for the determination of n, the relation

$$M = \frac{P_1 + P_2 + P_3}{n}\,s_1$$

where s_1 is the distance-apart of the two springs over an axle: and consequently we have

$$\frac{1}{n} = \frac{2\,G_2\,(x_2 - x_1) + (X + G_3)\,(x_3 - x_1)}{(P_1 + P_2 + P_3)\,s_1} \quad \ldots \quad (55)$$

As the loads on the wheels alter, so, of course, does the load on the rails; thus the pressure exerted by the inner wheels is $P_1 + p_1$, $P_2 + p_2$, $P_3 + p_3$; and that in the outer wheels is $P_1 + q_1$, $P_2 + q_2$, $P_3 + q_3$.

The values of p and q are determined as follows.

For the F. A. referring to Fig. 33

$$P_1\left(1 - \frac{1}{n}\right) + P_1\left(1 + \frac{1}{n}\right) - (P_1 + p_1) - (P_1 + q_1) = 0\,;$$

whence
$$p = -q \cdot$$
Also
$$(P_1 + p_1)\, s - \left(P_1 - \frac{P_1}{n}\right)(s - \epsilon) - P_1\left(1 + \frac{1}{n}\right)\epsilon - (G_1 - Y)\, x_1 = 0:$$
consequently,
$$p_1 = -\frac{P_1 \frac{s_1}{n} - (G_1 - Y)\, x_1}{s} \quad \dots \quad \dots \quad \dots \quad (56)$$
Similarly, for the M. A.,
$$p_2 = -q_2$$
and therefore
$$p_2 = -\frac{P_2 \frac{s_1}{n} - 2\, G_2\, x_2}{s}, \quad \dots \quad \dots \quad \dots \quad (57)$$
For the H. A.,
$$p_3 = -q_3$$
and therefore
$$p_3 = -\frac{P_3 \frac{s_1}{n} - (X + G_3)\, x_3}{s} \quad \dots \quad \dots \quad \dots (58)$$

If we insert in the above expressions the approximate values for a Normal Locomotive, viz. $s_1 = 1{\cdot}18^m$, $s = 1{\cdot}5^m$, and for Y, G_1, G_2, G_3, and X, the values given in § 25, then we obtain the following Table of the values of p_1, p_2, p_3, and implicitly, of q_1, q_2, q_3.

These values for p and q refer to a light-running locomotive.

Vehicle.	$R = 180^m$			$R = 300^m$			$R = 1000^m$		
	p_1	p_2	p_3	p_1	p_2	p_3	p_1	p_2	p_3
Passenger Loco.	$-{\cdot}16\,P_1$	${\cdot}24\,P_2$	$-{\cdot}08\,P_3$	$-{\cdot}15\,P_1$	$-24\,P_2$	$-{\cdot}08\,P_3$	$-{\cdot}14\,P_1$	${\cdot}25\,P_2$	$-{\cdot}10\,P_3$
Goods do.	$-{\cdot}15\,P_1$	${\cdot}19\,P_2$	$-{\cdot}03\,P_3$	$-{\cdot}15\,P_1$	${\cdot}19\,P_2$	$-{\cdot}04\,P_3$	$-{\cdot}13\,P_1$	${\cdot}22\,P_2$	$-{\cdot}08\,P_3$

For 2-axled vehicles, having F-and H-wheels of equal diam., we have
$$M = 0 : n = \infty\ ;$$
$$p_1 = -q = (G_1 - Y)\frac{x}{s}$$
$$p_3 = -q_3 = (X + G_3)\ \frac{x}{s}.$$

The values of p_1 and p_3, derived from the above two equations are, owing to their smallness, of no importance as regards the load on the rails.

For a 3-axled locomotive, also, according to the preceding Table, p_3 is insignificant whereas p_1 and p_2 are not so; and in curves of less than 1000^m $R.$ their average value is as follows :—

(a) for a Passenger locomotive: $p_1 = -{\cdot}15\,P_1$
$p_2 = \ {\cdot}24\,P_2$

(b) for a Goods locomotive : $p_1 = -{\cdot}15\,P_1$
$p_2 = \ {\cdot}19\,P_2$,

For the hauling locomotive the values of p_1 and p_2 are smaller, because Y, G_1, G_2, G_3, and X diminish as Z increases : consequently, M becomes smaller.

A further alteration of the wheel-loading occurs if the outer-rail *superelevation is not that corresponding to the velocity of locomotive* at the moment. Suppose a curve having a superelevation corresponding to a velocity v, is run over with a velocity V, and that $V > v$; then in each axle of the vehicle there is a centrifugal force (acting towards the convex side of the curve) of the magnitude, $2\,P\,\dfrac{V^2 - v^2}{g\,R}$;

Fig. 33

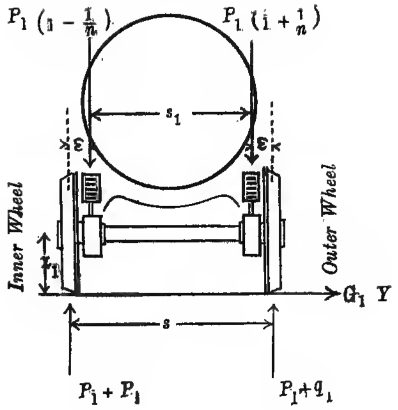


Fig. 34.

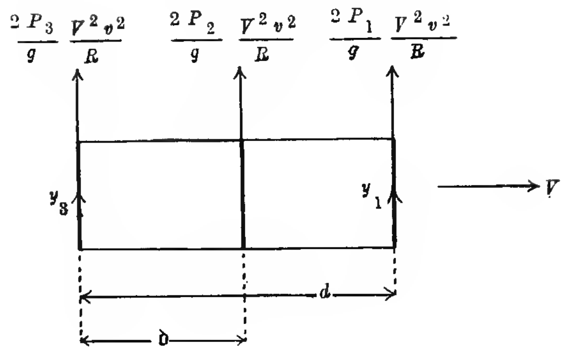
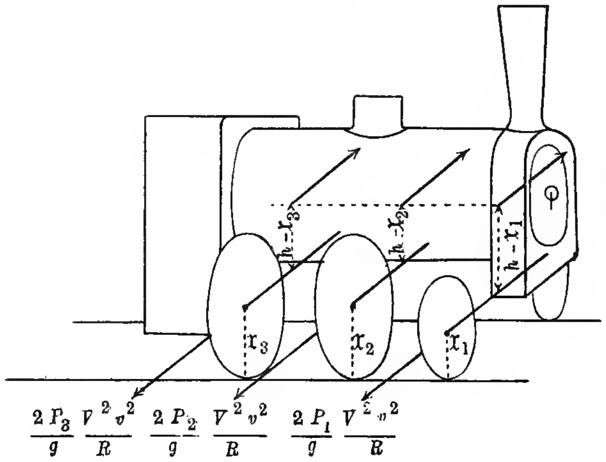


Fig. 35.



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This new force entirely alters that wheel-loading of the locomotive corresponding to the normal rail-superelevation. In addition, the forces acting in the end-axles are also affected. Referring to **Fig. 34**—let the forces arising in the F. and H. A. due to abnormal superelevation be denoted by y_1 and y_3.

Then we have

$$y_1 = \frac{2\,P_1}{g} \cdot \frac{V^2 - v^2}{R} + \frac{2\,P_2}{g} \cdot \frac{V^2 - v^2}{R} \cdot \frac{\mathfrak{d}}{d}$$

$$= P_1 \frac{2 + 2\,\frac{P_2}{P_1} \cdot \frac{\mathfrak{d}}{d}}{g} \cdot \frac{V^2 - v^2}{R}.$$

and
$$y_3 = P_3 \frac{2 + 2\,\frac{P_2}{P_3}\,\frac{d - \mathfrak{d}}{d}}{g} \cdot \frac{V^2 - v^2}{R}.$$

Whence we obtain

(*a*) for Passenger locomotive, $y_1 = \cdot 32\,P_1 \dfrac{V^2 - v^2}{R}$,

$$y_3 = \cdot 29\,P_3 \frac{V^2 - v^2}{R}.$$

(*b*) for Goods locomotive, $y_1 = \cdot 29\,P_1 \dfrac{V^2 - v^2}{R}$,

$$y_3 = \cdot 31\,P_3 \frac{V^2 - v^2}{R}.$$

For the 2-axled vehicle,

$$y_1 = y_3 = \frac{2\,P}{g} \frac{V^2 - v^2}{R} = cir.\ \cdot 2\,P \frac{V^2 - v^2}{R}.$$

A further result of abnormal superelevation occurs in the body of the locomotive as a torsion-moment M_1 such that

$$M_1 = \left(\frac{2\,P_1}{g} (h - \mathfrak{x}_1) + \frac{2\,P_2}{g} (h - \mathfrak{x}_2) + \frac{2\,P_3}{g} (h - \mathfrak{x}_3) \right) \frac{V^2 - v^2}{R}$$

wherein,—referring to **Fig. 35**,— h is the height of the C. G. of locomotive above the rails.

This moment super-loads the springs on the outside rails by the amounts $\frac{P_1}{n_1}, \frac{P_2}{n_2}, \frac{P_3}{n_3}$, and unloads those on the opposite side by the same amount.

Consequently, $$M_1 = \frac{P_1 + P_2 + P_3}{n_1} s_1$$

and we obtain

$$\frac{s_1}{n_1} = \left(\frac{2\,h}{g} - 2 \frac{P_1\,\mathfrak{x}_1 + P_2\,\mathfrak{x}_2 + P_3\,\mathfrak{x}_3}{g\,(P_1 + P_2 + P_3)} \right) \frac{V^2 - v^2}{R} \qquad \ldots \qquad \ldots \quad (59)$$

And similarly to the Eqns. 56, 57, 58, for the increase of loading of the inside wheels due to M_1, we have

$$\mathfrak{p}_1 = - \frac{P_1 \frac{s_1}{n_1} + y_1\,\mathfrak{x}_1}{s}$$

$$\mathfrak{p}_2 = - \frac{P_2 \frac{s_1}{n_1}}{s}$$

$$\mathfrak{p}_3 = - \frac{P_3 \frac{s_1}{n_1} + y_3\,\mathfrak{x}_3}{s}.$$

For 2-axled vehicles we have

$$\mathfrak{p} = - \cdot 20 \frac{V^2 - v^2}{R} P \frac{h}{s}.$$

Assuming $h = 1{\cdot}5^m$, we obtain from the above equations,

(a) for the forwards-running Passenger locomotive :—

$$\mathfrak{p}_1 = -\cdot 22\, P_1 \frac{V^2 - v^2}{R}.$$

$$\mathfrak{p}_2 = -\cdot 10\, P_2 \frac{V^2 - v^2}{R}.$$

$$\mathfrak{p}_3 = -\cdot 27\, P_3 \frac{V^2 - v^2}{R}.$$

(b) for the forwards-running Goods locomotive :—

$$\mathfrak{p}_1 = -\cdot 23\, P_1 \frac{V^2 - v^2}{R}.$$

$$\mathfrak{p}_2 = -\cdot 11\, P_2 \frac{V^2 - v^2}{R}.$$

$$\mathfrak{p}_3 = -\cdot 25\, P_3 \frac{V^2 - v^2}{R}.$$

If, as is very commonly the case, all curves of radii below 300^m are superelevated according to the formula $\frac{45}{R}$, (which is based on an assumed velocity $v = 17{\cdot}12\,{}^m/_{sec.}$) and if such curves are traversed by a Goods engine running with the velocity $V = 6\,{}^m$, then $\mathfrak{p}_1, \mathfrak{p}_2, \mathfrak{p}_3$ assume the following values :—

$R = 300^m$	$R = 400^m$	$R = 500^m$	$R = 600^m$	$R = 800^m$	$R = 1000^m$
$\mathfrak{p}_1 = \cdot 20\, P_1$	$\cdot 15\, P_1$	$\cdot 12\, P_1$	$\cdot 10\, P_1$	$\cdot 07\, P_1$	$\cdot 06\, P_1$
$\mathfrak{p}_2 = \cdot 09\, P_2$	$\cdot 07\, P_2$	$\cdot 06\, P_2$	$\cdot 05\, P_2$	$\cdot 04\, P_2$	$\cdot 03\, P_2$
$\mathfrak{p}_3 = \cdot 22\, P_3$	$\cdot 17\, P_3$	$\cdot 13\, P_3$	$\cdot 11\, P_3$	$\cdot 08\, P_3$	$\cdot 07\, P_3$

Let these alterations in wheel-load be combined with those—for the Goods locomotive—of p_2, p_3 (and q_1), and we obtain the wheel-pressures for the light-running Goods locomotive exhibited in the following Table.

Wheel.	$R = 300^m$	400^m	500^m	600^m	800^m	1000^m
Outer F-wheel ...	$P_1 + q_1 + \mathfrak{q}_1 = \cdot 95\, P_1$	$1\, P_1$	$1{\cdot}03\, P_1$	$1{\cdot}05\, P_1$	$1{\cdot}08\, P_1$	$1{\cdot}09\, P_1$
Inner M-wheel ...	$P_2 + p_2 + \mathfrak{p}_2 = 1{\cdot}28\, P_2$	$1{\cdot}26\, P_2$	$1{\cdot}25\, P_2$	$1{\cdot}24\, P_2$	$1{\cdot}23\, P_2$	$1{\cdot}22\, P_2$

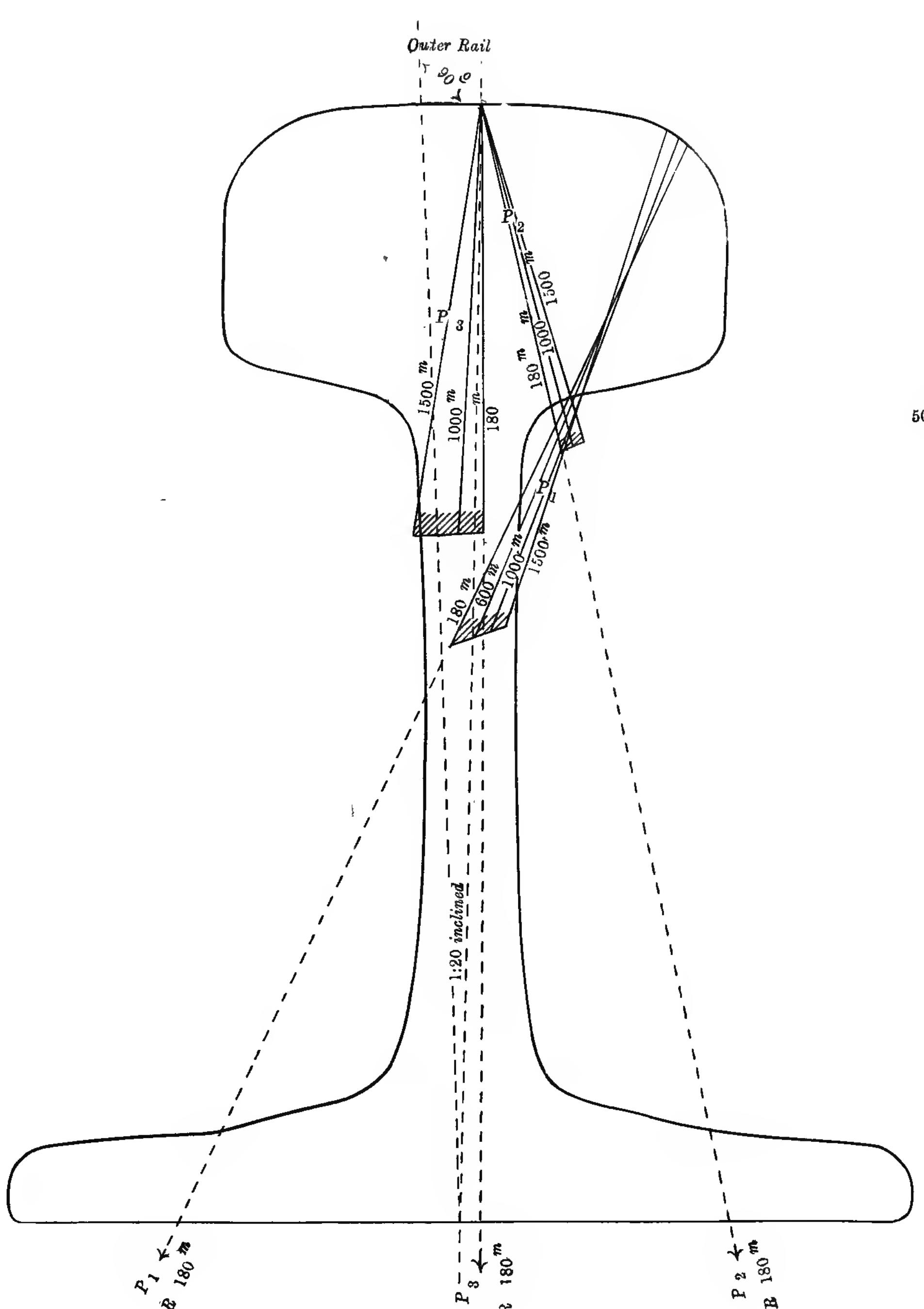
In the above $\mathfrak{q}_1$ indicates the increase of load, corresponding to $\mathfrak{p}_1$ of the outer F-wheel.

These wheel-loadings also occur in an engine at the head of a train if, when traversing a curve, no traction be exerted. But if the curve is on the level, or on a rising gradient, then q_1 and p_2 and consequently, also, the variations in the loads on the wheels, are smaller the greater the T. F. is.

Whence we see that in *normally, i.e.* correctly, superelevated curves the inner M-wheel, and consequently the rail, is especially heavily loaded. Thus for the light-running Passenger locomotive, the load is $1{\cdot}24 \times 6100$ *kg.* = 7560 *kg.* say; and for the light-running Goods locomotive, it is $1{\cdot}19 \times 6230 = 7400$ *kg.*

Further, the expressions for $\mathfrak{p}_1, \mathfrak{p}_2, \mathfrak{p}_3$, show that irregular superelevation of curves affects the loading of the F- and H-wheels almost twice as much as it does that of the M.-wheels.

And finally, it depends on the ratio of V to v whether the maximum loading occurs at the inner M-wheel or at the outer F-wheel.



Scale for Forces

$50\ {}^{m}/_{m} = P_1,\ P_2,\ P_3$

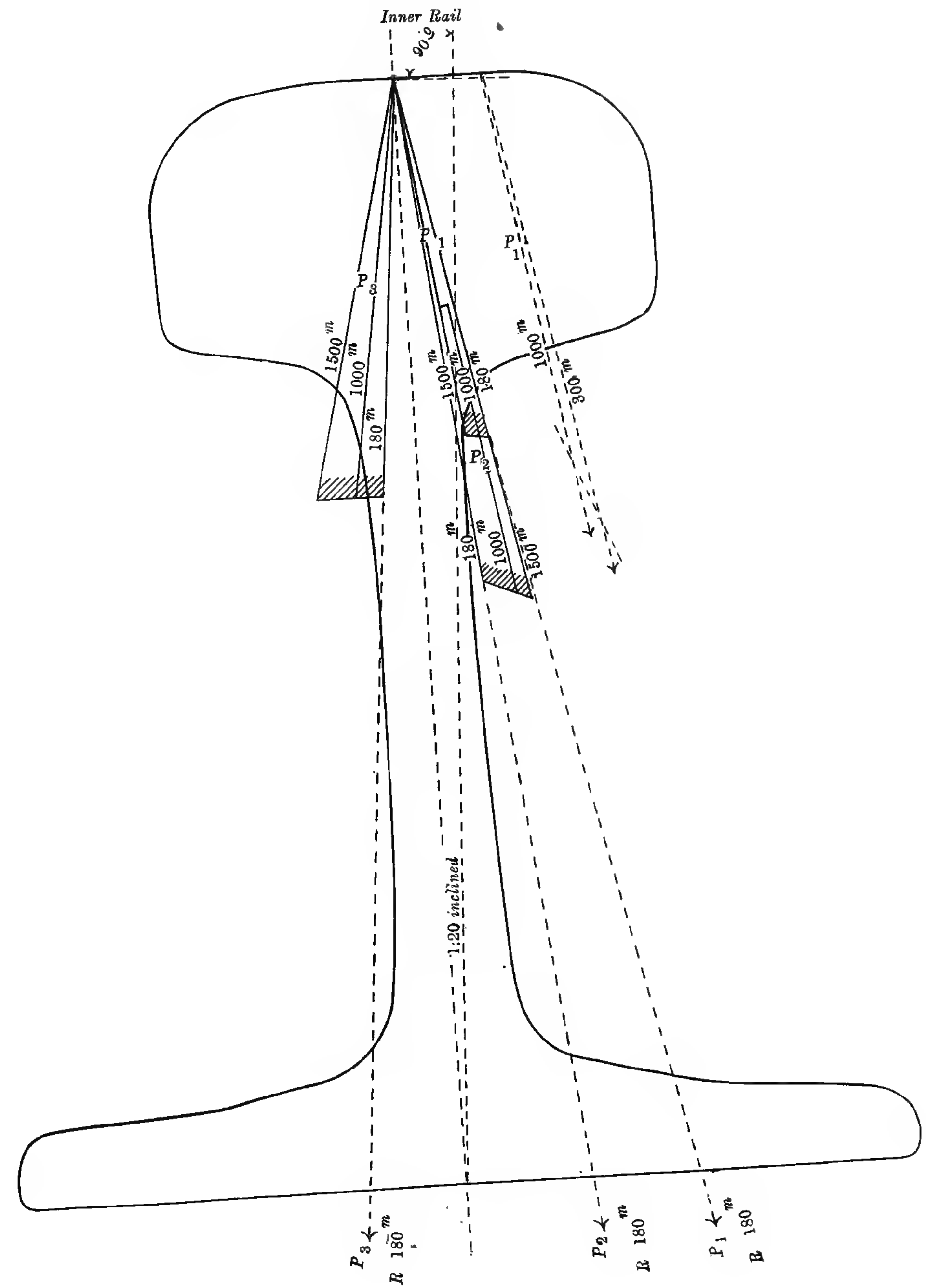


Fig. 36.

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§ 28.

Graphic representation of the Action on the rails of a light-running Goods Locomotive.

With a view to show how the rails are stressed by a light-running Goods locomotive the forces exerted by the wheels in normally superelevated curves of 180^m, 1000^m, and 1500^m rad. are represented graphically in **Fig. 36** in magnitude, direction, and position. In the Fig. it is assumed that the cant of the rails is $\frac{1}{20}$, and that it remains unaltered by any action of the wheels; accordingly the inner F-wheel and the wheels of the M. and H. A. are assumed to rest on the middle of the rail-head.

The resultant wheel-pressures are respectively indicated according to their origins by P_1, P_2, P_3; and further the radii are indicated of the curves in which they occur.

The lengths of the lines representing these resultants are expressed by taking the wheel-pressures P_1, P_2, P_3, of the locomotive when standing at rest as unity, and representing each of these wheel-pressures by a line of 50^{mm} in length.

Thus, for example, the length of the line representing the resultant pressure of the outer F-wheel in a curve of 180^m R. is $65^m/_m$ and this resultant has, consequently, the magnitude $\frac{65}{50} P_1 = 1{\cdot}3\, P_1$.

If the Goods locomotive traverse a curve superelevated according to the formula $h = \frac{45}{R}$, then, from § 27, there is a considerable alteration in the distribution of load *only* for the F. and H. A. in *sharp* curves; thus the inner-wheels of these axles in curves of 300^m to 1000^m rad. become more heavily loaded by $\mathfrak{p}_1 = {\cdot}2\, P_1$, and $\mathfrak{p}_3 = {\cdot}22\, P_3$ up to ${\cdot}06\, P_1$ and ${\cdot}07\, P_3$, respectively; and the outer wheels are unloaded by the same amounts.

The resultant at the inner F-wheel in this case is indicated by the dotted-line at the side.

Since every wheel-pressure which does not pass through the vertical axis of the cross-section of the rail causes a torsion of the rail, the Fig. shows that the head of the outer-rail is pressed outwards by the F-wheel, and inwards by the M-wheel, but by the H-wheel it is almost completely maintained in the position in which this wheel happens to catch it at the time.

Thus the M-wheel opposes the thrusting-asunder action of the F-wheel; and, as appears from the Fig., does so very energetically, since the directions of the forces of these two wheels cut the base of the rail at almost equal distance from the opposite sides. Owing to this difference of direction these forces exercise a severe torsive action—increasing with the elasticity or yielding of the rail-substructure—of that portion of the outer-rail lying between these two wheels; and they tend consequently to bring about a loosening of the joint-fastenings.

With the 3-axled locomotive, the action on the inner-rail is different. Here the F- and M-wheels both tend to thrust asunder the rails, while the H-wheel tends to bring them together. Consequently, there is not such a severe stressing of the joint-fastenings of the inner-rail as there is of those of the outer-rail between the outer F- and M-wheels. But, on the other hand, the moment tending to cant the inner-rail is greater than at the outer; and is, accordingly the more dangerous one to the safety of travel of the vehicle.

In the construction of Fig. 36 it is assumed, that the inclination of the rails, *i.e.* $\frac{1}{20}$, is unaltered by the action of the locomotive. This is not exactly the case, because the forces produced by the wheels do not intersect the rail-base in its middle; and consequently a rotation more or less great of the rail about its longitudinal axis, depending on the elasticity or yielding of the rail-substructure, takes place.

In this connexion it is to be noted that this canting of the rail favours its stability because the points of application of the resultants of the forces have then a more favourable position relatively to the foot of the rail, since they are displaced further towards the centre of the track.

CHAPTER IV.

THE BEHAVIOUR OF A 4-WHEELED VEHICLE IN FLAT CURVES AND STRAIGHTS. RAIL WEAR.

§ 29.

The Motion of Vehicles in flat curves.

We see from Eqn. 16 that Y diminishes as $R\sigma$ increases: and consequently it is evident that there are values of R and σ which will make $Y = 0$. But these values cannot be obtained from that equation because this latter is based on the radial position of the H. A.: and this position, as we have already seen from Eqn. 29, does not occur in flat curves, where $Y < G$. In such curves the vertical axis of the vehicle's horizontal rotation lies somewhere between the F. and H. Axles; and, from the foregoing discussion, it results that if with normal superelevation Y is to be $< G$, the apexes of the rolling-cones of both axles relatively to the centre M of the curve must have the position shown in **Fig. 37**.

From this Fig. it follows that in flat curves

$$Y = 0,$$

if

$$K_1 s + K_2 s = + 2 G_1 d = - 2 G_2 d.$$

This equation holds when the two outer-wheels of both axles run with uniform pressure against the outer-rail; and we have in that case the minimum value of $R\sigma$ which makes $Y = 0$.
We obtain, then, for this minimum:—

$$G_1 = - G_2; \quad \rho_1 = \rho_2; \quad K_1 = K_2; \quad m = q = \frac{d}{2}:$$

and further

$$K s = G d.$$

Accordingly, from Eqns. 13 and 14, putting $b = 0$,
we have

$$\frac{\frac{s}{2}\left(\frac{R\sigma}{n x} - s\right)}{\sqrt{\frac{d^2}{4} + \frac{1}{4}\left(\frac{R\sigma}{n x} - s\right)^2}} = \frac{\frac{d^2}{2}}{\sqrt{\frac{d^2}{4} + \frac{1}{4}\left(\frac{R}{n x} - s\right)^2}}:$$

and further

$$R\sigma = n x\left(\frac{d^2}{s} + s\right) \quad \dots \quad \dots \quad \dots \quad (60)$$

Accordingly, $Y = 0$, when $R\sigma$, R, and σ, have the values given in the following Table.

Vehicle.	$s = 1{\cdot}5^m$				
				R	
	d	$n x$	$R\sigma$	$\sigma = {\cdot}01$	$\sigma = {\cdot}025$
	m	m	m	m	m
Passenger cars	5	10	182	18200	7280
Goods waggons	4	10	122	12200	4880
Sec.-Line locos.	2·25	11	62	6200	2480
Truck 8-wheeled vehicle	1·3	10	26	2600	1040

If we take the curve-radii as abcissœ, and the corresponding values of σ as ordinates, the products $R\sigma$ may be represented graphically — as in **Fig. 38**. Each point in these curves corresponds thus to a pressure $Y = 0$.

Fig. 37.

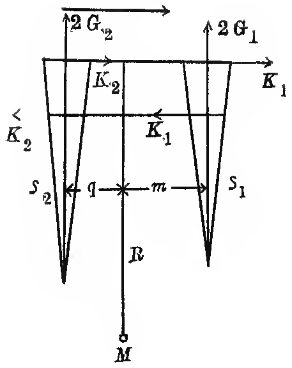


Fig. 38.

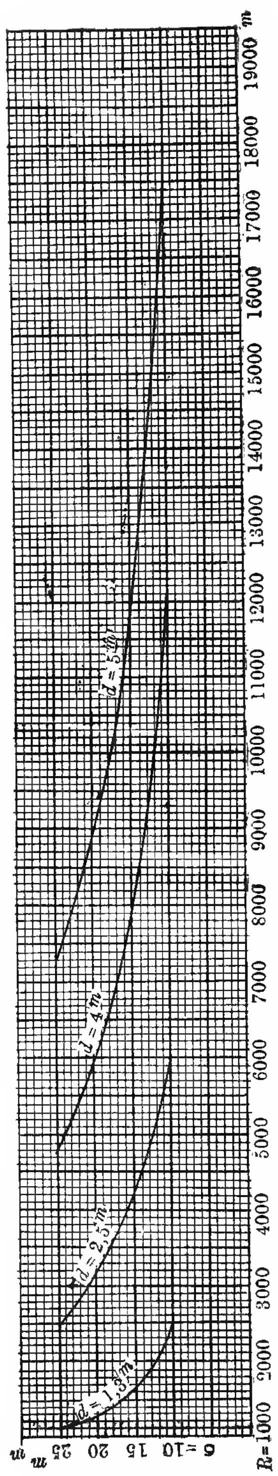
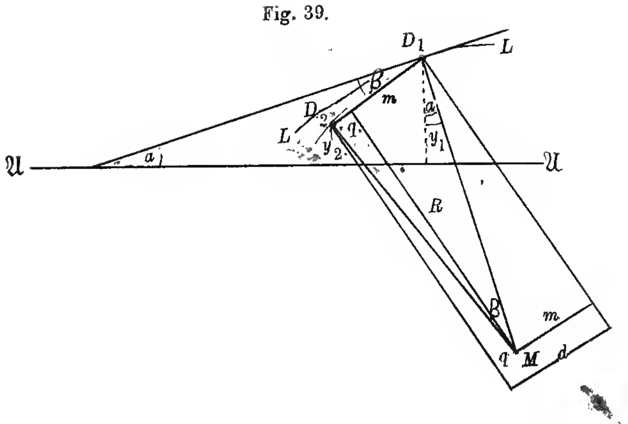


Fig. 39.



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From Eqn. 60 it follows that the smallest R. of the curve in which a vehicle can run without curve-pressure increases not only with d but also with n and x; and for $n = \infty$ (cylindrical wheels) is infinitely great. A vehicle with cylindrical wheels, consequently, when running through a flat curve will always exert a lateral pressure Y against the outer-rail — unless special external forces are introduced to counteract the action of Y.*

In Eqn. 60, σ represents *twice* the amount of the possible displaceability of the middle of axle from the middle of the track. Representing this displaceability in general by σ', and, taking a case where $R\sigma > n\,x\left(\frac{d^2}{s} + s\right)$ then the vehicle will run through the curve without lateral pressure, if at both axles,

$$\sigma' = \frac{n\,x}{2R}\left(\frac{d^2}{s} + s\right).$$

The displacement σ' consequently decreases as R increases, and in the straight is *nil.*

If in curves for which $R\sigma > n\,x\left(\frac{d^2}{s} + s\right)$ it happens that from the condition of the track or from the influence of the adjacent vehicles, or otherwise, a force arises in addition to K_1, K_2, G_1, G_2, then the normal or theoretic circular motion of the vehicle in the track will be disturbed, and the well-known serpentine swing-movement will ensue.

The radii given in the Table, accordingly, supply a criterion for the minimum curvature which the track should have in order that the vehicle may preserve the smooth travel of curvilinear motion.

In the foregoing it is tacitly assumed that the axles remain constantly parallel to one another. If the axles were originally parallel to start with, this is really so, for the moments Ks in the wheel-pairs are in the same sense; and consequently, owing to the unavoidable clearance of the axle-bearings in their guides, the direction in which the axles are free to rotate relatively to the underframe is in both cases the same.

§ 30.

The Motion of Vehicles in the Straight.

In straights, where the lateral pressures Y and X arising from curvature do not exist, the forces G and K acting at the wheel-treads become the predominant factors determining the motion of the vehicle in so far as the clearance and the influence of the coupling with the adjacent vehicles permits.

---

* From Eqn. 60 we obtain at once

$$R = \left(\frac{d^2}{s^2} + 1\right)\rho,$$

where ρ is height of the rolling-cones of both wheel-pairs. If we take 2 wheel-pairs, identical in all respects, each of which if left freely to itself would describe on a horizontal plane a circle of radius ρ, and if we place them parallel to one another in a frame, at a fixed distance, d, apart, then this compound arrangement will roll in a curve of radius $= \rho\left(\frac{d^2}{s^2} + 1\right)$ — assuming the rolling-perimeters of each wheel-pair to be distant apart s from each other.

If, for example, $\rho = 1^m$, $\frac{d}{s} = 2{\cdot}5$;

then $R = 1\,(2{\cdot}5^2 + 1) = 7{\cdot}5^m$.

An experiment of this kind was carried out by McDowell and described in the (London) "Engineer," and republished therefrom in the "Wochenblatt des oesterreichischen Ingenieur-und Architeckten-Vereins": 1876, p. 331. This experiment, however, in contradiction to what we have stated above, and presumably in consequence of defective observation, is supposed to have shewn that the path described by such an arrangement of 2 wheel-pairs is a straight line, and not a curve.

In **Fig. 39** let 𝔄 𝔄 represent the axis of a straight piece of track; D_1 the middle of the F. A.; D_2 the middle of the H. A.; $L\,L$ the path of the point D_1; $y_1\,y_2$ the distances of the points D_1, D_2, from 𝔄 𝔄, respectively; and let R, m, q, β, α, have the meanings given in the Fig. Then for the **F. A**,

$$K_1 = f\,P \frac{\frac{1}{2}\left[\frac{R\,2\,y_1}{n\,x} - s\right]}{\sqrt{m^2 + \frac{1}{4}\left(\frac{R\,2\,y_1}{n\,x} - s\right)^2}}$$

and

$$G_1 = f\,P \frac{m}{\sqrt{m^2 + \frac{1}{4}\left(\frac{R\,2\,y_1}{n\,x} - s\right)^2}}$$

For the **H. A**,

$$K_2 = f\,P \frac{\frac{1}{2}\left(\frac{R\,2\,y_2}{n\,x} - s\right)}{\sqrt{q^2 + \frac{1}{4}\left(\frac{R\,2\,y_2}{n\,x} - s\right)^2}}$$

and

$$G_2 = -f\,P \frac{q}{\sqrt{q^2 + \frac{1}{4}\left(\frac{R\,2\,y_2}{n\,x} - s\right)^2}}$$

Now for a *single* moving vehicle,

$$K_1\,s + K_2\,s = 2\,G_1\,d = -2\,G_2\,d:$$

and therefore $\qquad G_1 = -\,G_2$

and accordingly,

$$K_1 + K_2 = \frac{2\,d}{s} G_1 = -\frac{2\,d}{s} G_2.$$

Consequently, from the above equations,

$$q = \frac{s}{2\,d}\left(\frac{R\,2\,y_2}{n\,x} - s\right) \quad \ldots \quad \ldots \quad \ldots \quad (61)$$

$$m = \frac{s}{2\,d}\left(\frac{R\,2\,y_1}{n\,x} - s\right) \quad \ldots \quad \ldots \quad \ldots \quad (62)$$

$$G_1 = f\,P \frac{\frac{s}{2}}{\sqrt{\left(\frac{s}{2}\right)^2 + \left(\frac{d}{2}\right)^2}}$$

and

$$K_1 = K_2 = f\,P \frac{\frac{d}{2}}{\sqrt{\left(\frac{s}{2}\right)^2 + \left(\frac{d}{2}\right)^2}}$$

It may be remarked in passing, that these expressions show that the 4 sliding-resistances, $f\,P$, intersect one another in the centre of, and underneath, the vehicle.

From Eqns. 61 and 62 we have

$$R\,(y_1 + y_2) = n\,x\left(\frac{d^2}{s} + s\right) \quad \ldots \quad \ldots \quad \ldots \quad (63)$$

and from Fig. 39

$$y_1 - y_2 = d \sin\,(\alpha + \beta) = d\,(\tan\alpha + \tan\beta) \cos\alpha \cos\beta.$$

Or, since we may put

$$\cos\alpha = \cos\beta = 1,$$

then

$$y_2 = y_1 - d\,(\tan\alpha + \tan\beta).$$

Putting $\tan\alpha = \frac{d\,y}{d\,x}$, and $\tan\beta = \frac{m}{R}$ — where m is given by Eqn. 62—

then

$$y_2 = y_1 - d\,\left[\frac{d\,y}{d\,x} + \frac{s}{2\,d}\left(\frac{2\,y_1}{n\,x} - \frac{s}{R}\right)\right]$$

combining this with Eqn. 63, and putting $R = -\frac{1}{\frac{d^2y}{dx^2}}$

we obtain

$$\frac{d^2 y}{d x^2} \frac{n x\left(\frac{d^2}{s} + s\right) - \frac{s^2}{2}}{2 - \frac{s}{n x}} - \frac{d y}{d x} \frac{d}{2 - \frac{s}{n x}} + y = 0 \quad \ldots \quad \ldots \quad (64)$$

and finally, referring to the system of co-ords. of **Fig. 40,**

$$y = h e^{\frac{a x}{2 b}} \left[\cos x \sqrt{\frac{1}{b} - \frac{a^2}{4 b^2}} - \frac{a}{\sqrt{4 b - a^2}} \sin x \sqrt{\frac{1}{b} - \frac{a^2}{4 b^2}} \right] \ldots \quad (65)$$

Here the Y-axis passes through the crest or apex of the wave; and $h =$ the height of wave when $x = 0$ and a and b have the following values:—

$$a = \frac{d}{2 - \frac{s}{n x}}$$

$$b = \frac{n x\left(\frac{d^2}{s} + s\right) - \frac{s^2}{2}}{2 - \frac{s}{n x}}.$$

From Eqn. 65,

$$y = 0$$

when

$$x = \frac{2 b}{\sqrt{4 b - a^2}} \arctan \sqrt{\frac{4 b - a^2}{a^2}}:$$

and the wave-length

$$L = 2 \pi \frac{2 b}{\sqrt{4 b - a^2}}:$$

or, sufficiently exactly,

$$L = 2 \pi \sqrt{b} = 2 \pi \sqrt{\frac{n x\left(\frac{d^2}{s} + s\right) - \frac{s^2}{2}}{2 - \frac{s}{n x}}} \quad \ldots \quad \ldots \quad (66)$$

Whence we obtain the following Table:—

$s = 1{\cdot}5^m$; $n x = 10$.	
d	L
m	m
5	62·2
4	50·6
1·3	23·1

Putting in Eqn. 65,

$$x = z L = z\, 2 \pi \frac{1}{\sqrt{\frac{1}{b} - \frac{a^2}{4 b}}}.$$

we obtain for the height of the z^{th} wave-crest above the X-axis,

$$h_z = h\, e^{\frac{z\, a\, \pi}{\sqrt{b - \frac{a^2}{4}}}}$$

or sufficiently accurately,

$$h_z = h\, e^{z\, \pi \frac{a}{\sqrt{b}}} \quad \ldots \quad \ldots \quad \ldots \quad (67)$$

And hence putting $h_z = \frac{\sigma}{2}$, the number of oscillations or sinuosities, starting from the initial height h, down to the impinging of the F. A., is

$$z_{\max} = \frac{\sqrt{b}}{a\,\pi} \text{ nat. log } \frac{\sigma}{2\,h}\,.$$

Whence it follows that each displacement, however small, of the middle of the axle from the centre of track results in a wavy course of the vehicle, of which the amplitude increases with each swing of the axle until it attains its maximum value in that of the clearance.

For example, let

$d = 4^m$, $\sigma = {\cdot}0175^m$.

Then for $h = {\cdot}00175$, $z/\max = 1{\cdot}9$;

and for $h = {\cdot}000875$, $z/\max = 2{\cdot}7$.

Thus in the first of the above cases the vehicle's F. A. would impinge against the rail before the second wave-length had been completely described: and in the second case, before the third wave-length.

If δ_1 be the angle at which the above curve crosses the axis of the track,

then $$\tan \delta_1 = \frac{dy}{ax} = -\frac{a}{\sqrt{b}}\, e^{\frac{a}{\sqrt{b}} \frac{\pi}{4}}$$

and thus for

h	$d = 5^m$	$d = 4^m$	$d = 1{\cdot}3^m$
$= {\cdot}003$	$\delta_1 = 1'\,20''$	$\delta_1 = 1'\,40''$	$\delta_1 = 3'\,30''$
$= {\cdot}00075$	$\delta_1 = 3'\,50''$	$\delta_1 = 4'\,40''$	$\delta_1 = 9'\,30''$

At high velocities the F. A. after impact with the rail—owing to the elastic rebound of the rail in returning to its normal position, and to the compression of the laterally springing axle-horn—may be so forcibly thrown back again towards the centre of track that its path after leaving its rail may form an angle with the axis of track.

In **Fig. 41**, let $\mathfrak{M}$ be the point of impact or re-spring at which the F. A. under these circumstances begins its return movement: then the path up to its crossing the axis of track is shortened by the piece $M'\,\mathfrak{M}$, and the path to be described between the points of impact, $\mathfrak{M}$ and $\mathfrak{M}'$ forms only a part of the half wave-length $\frac{L}{2}$

Now the angle at which the F. A. after its last impact departs from the rail increases with the violence of the impact; and this violence, with the velocity; consequently, the distance between the impacts diminishes as the velocity increases. *

* M.M. von Weber experimenting with a goods-waggon having perfectly formed wheel-tires, by allowing it to run freely down of itself on an incline in first-class condition of 1: 30, on the Chemnitz-Risaer Railway, found that the distance apart of the points of impact at a velocity of one mile, was 16^m, and with a velocity of 5 miles, it was 4^m. von Weber: "*Die Technik des Eisenbahn-Betriebs*:" p. 134.

Fig. 40.

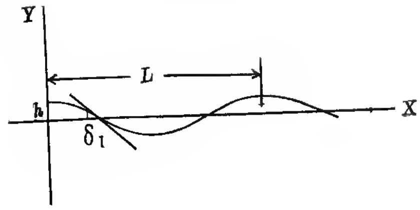


Fig. 41.

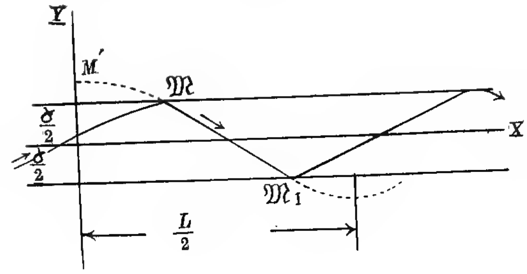


Fig. 42.

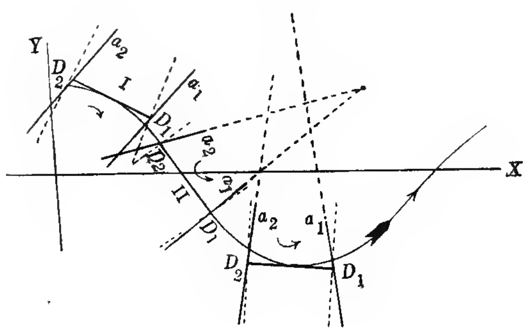


Fig. 43.

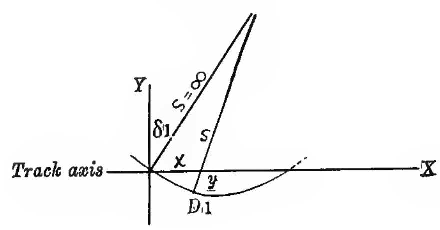
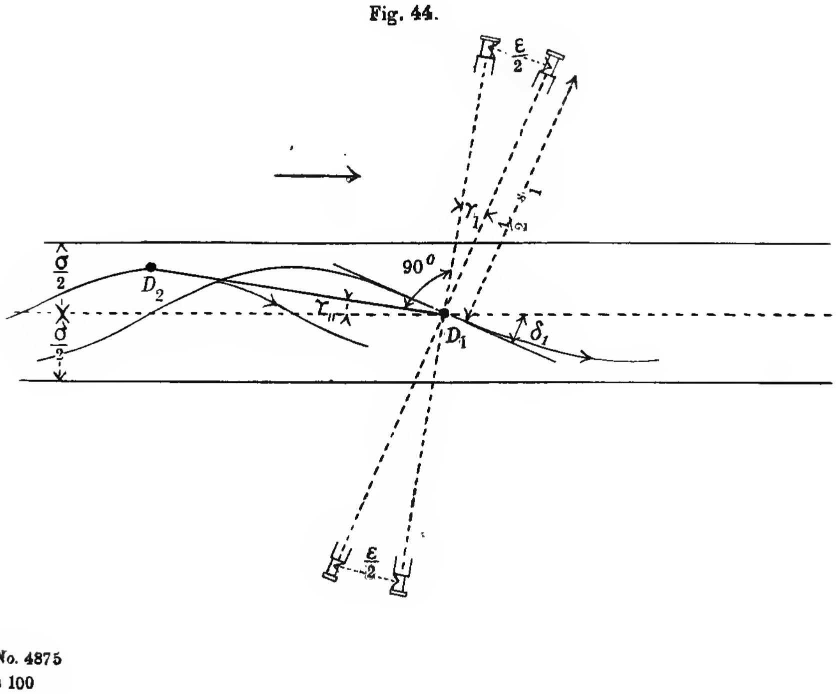
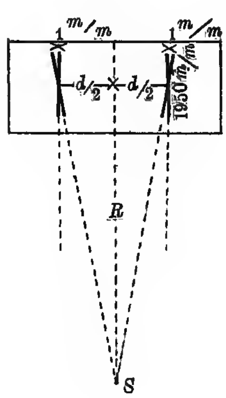


Fig. 44.



Fg. 45.



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Under the effect of the blow at the point of impact, which blow varies with the velocity of vehicle and the angle at which the impact occurs, the impinging wheel mounts on the rail to a height depending on the shape of the rail-head and on that of the hollow of the wheel-tire, the rail at the same time yielding elastically; and the vehicle in consequence of the centrifugal pressure acting on it, for the instant, rotates about the impinging wheel. In this way at the point of contact an increase takes place of the curvature of the path of the F. A. and therefore, of the force G at the inner F-wheel, and a corresponding widening of track at this wheel, due to the yielding of the inner-rail. At the H. A., meanwhile, both rails are sprung out of line in the direction of the impact.

The assumption made in deriving the Eqn. 66 for L—that the vehicle had been turned-out of the workshop with perfectly parallel axles, and that their parallelism was maintained under all circumstances—does not hold exactly for motion in the straight, because the axle-journals are not absolutely rigidly fixed in their bearings; and the bearings and boxes can execute small lateral movements of their own; the moments $K_1 s$ and $K_2 s$ are able, therefore, to turn the axles through an angle out of their normal middle-position.

The manner in which the moments $K_1 s$, $K_2 s$, can change the position of the axle relatively to the under-frame, is exhibited in **Fig. 42.** in which X, Y, D_1, and D_2 have their previous meanings as in Figs. 40 and 41 ; and I, II, indicate the position of the vehicle relatively to the axis of the track; a_1, a_2, the axles of the 2 wheel-pairs; the dotted lines passing through D_1 D_2 are verticals to the vehicle-axis D_1 D_2.

In the I-position, in which D_1 and D_2 lie above the track-axis, and K_1 and K_2 have thus the same direction, the axles a_1 and a_2 are turned towards the same side relatively to the dotted verticals, and are assumed to be parallel. When this state of things occurs, it is maintained until D_1 reaches the X-axis. But at that instant the direction of K_1 changes; and, consequently, the F. A. begins to return to its normal position and, on the further advance of the vehicle to the opposite side, to place itself obliquely to the axle $D_2 . D_1$.

During the passage of the F. A. from the position I into the position II, the value of K_1 does not depend on the value of y_1 as given by the expression on page 58, but upon the displacement of this axle from its normal position; viz., on the lateral movement of the axle-boxes in their guides and on the compression of the bearing-springs due to the lateral movement of the axle-boxes against the bearing springs.

K_1 will $= 0$ only at the instant in which the axle passes through the *normal* position.

If K_1 immediately after the passage of the point D_1 through the X-axis became *nil* and remained so, then the F. A. at any moment would rotate about the apex of its rolling-cone, and D_1 would describe a curve, which—from the condition

$$-\frac{1}{\dfrac{d^2 y}{d x^2}} = \rho = \frac{n r s}{2 y},$$

and with reference to **Fig. 43**—is represented by

$$y = -\tan \delta_1 \sqrt{\frac{n r s}{2}} \sin x \sqrt{\frac{2}{n r s}}. \quad \dots \quad \dots \quad (68)$$

This curve cuts the X-axis in points such that

$$\sin x \sqrt{\frac{2}{n r s}} = 0;$$

viz. for $x = o$, $x = \pi \sqrt{\frac{n r s}{2}}$, $x = 2 \pi \sqrt{\frac{n r s}{2}}$, etc.

The wave-length of this sine-curve is thus independent of δ_1 and is

$$l = 2 \pi \sqrt{\frac{n r s}{2}} = 6{\cdot}28 \sqrt{\frac{10 \times 1{\cdot}5}{2}} = 17{\cdot}2^m *$$

---

* M.M. von Weber found by experiment 16^m as the value of l for a velocity of 1 mile: "*Technik des Eisenbahn-Betriebes,*" p. 134.

Consequently, it is considerably smaller than the length L obtained from Eqn. 66.
For the height of wave we have

$$2\, y_{\max.} = 2 \tan \delta_1 \sqrt{\frac{n\, r\, s}{2}}\,.$$

The angle δ_1 is made up of the angle of rotation-displacement of the axle from its normal position, and of the inclination of the vehicle-axis to the axis of the track; and—referring to **Fig. 44**—is $\delta_1 = \gamma_{\text{I}} + \gamma_{\text{II}}$: or if D_2 lies on the opposite side of the track-axis—in Fig. 44 it lies under it—then

$$\delta_1 = \gamma_{\text{I}} - \gamma_{\text{II}}.$$

We obtain thus

$$2\, y_{\max} = 2 \tan (\gamma_{\text{I}} \pm \gamma_{\text{II}}) \sqrt{\frac{nrs}{2}}:$$

and from this, accordingly, it follows that the height of wave $2\, y_{\max.}$ increases with the angle of deviation or swing γ_{I}.
Also,

$$\tan (\gamma_{\text{I}} \pm \gamma_{\text{II}}) = \frac{y_{\max}}{\sqrt{\dfrac{n\, r\, s}{2}}}\,.$$

If $y_{\max}$ becomes = half the clearance, then

$$\tan (\gamma_{\text{I}} \pm \gamma_{\text{II}}) = \frac{\sigma}{2} \quad \frac{1}{\sqrt{\dfrac{n\, r\, s}{2}}}\,. \qquad \ldots \qquad \ldots \quad (69)$$

Should the clearance of the axle-bearings permit a greater angle γ_{I}, that is, if

$$\tan (\gamma_{\text{I}} \pm \gamma_{\text{II}}) > \frac{\dfrac{\sigma}{2}}{\sqrt{\dfrac{n\, r\, s}{2}}}\,,$$

then $2\, y_{\max} > \sigma$, and the F. A. wheels run only intermittently against the rails, and the serpentine motion is thus destroyed. Under these circumstances, the crossings by D_1 of the middle of track are less apart than $\frac{l}{2}$ from one another.

Numerical Illustration:—

From Eqn. 69 we have for $n\, r = 10$, $s = 1{\cdot}5^m$, $\sigma = {\cdot}0175$

$$\tan (\gamma_{\text{I}} \pm \gamma_{\text{II}}) = \frac{{\cdot}0175.}{2} \quad \frac{1}{\sqrt{\dfrac{10 \times 1{\cdot}5}{2}}}:$$

whence $\gamma \pm \gamma_{\text{II}} = 11'\, 20''$.
The maximal value of γ_{II} is given by

$$\tan \gamma_{\text{II}} = \frac{\sigma}{2d} = \frac{{\cdot}0175}{2 \times 4^m},$$

or

$$\gamma_{\text{II}} = 8'\, 26''.$$

With this value of γ_{I}, $2\, y_{\max} = \sigma = {\cdot}0175$,
if

$$\gamma_{\text{I}} = 11'\, 20'' \mp 8'\, 26''$$

or

$$\gamma_{\text{I}} = 2'\, 54'', \text{ and } 19'\, 46''.$$

In order that the occurrence of this deflection-angle γ_{I} may be possible, the axle-journals must have an angular play (Fig. 44) $\epsilon = s_{\text{I}} \sin \gamma_{\text{I}}$, where s_{I} is the distance apart of the journals, c to c.

In vehicles $s_{\text{I}} = 1{\cdot}956^m$, and thus $\epsilon = 1{\cdot}956 \sin 2'\, 54'' = {\cdot}0017^m$ for the position of D_1 above the axis of the track; and $\epsilon = 1{\cdot}956 \sin 19'\, 54'' {\cdot}0012^m$, when it is below it. Fig. 44.)

If $\gamma_{\text{II}} = 0$, namely if D_2 lies in (Fig. 44 – the middle of the track, then, when $2\gamma_{\max} = {\cdot}075$,

$$\epsilon = 1{\cdot}956 \sin 11'\, 20'' = {\cdot}0065^m.$$

From the above it follows that when $\epsilon = 1{\cdot}7^{mm}$, the height of wave can never exceed the value $2\gamma_{\max} = 0{\cdot}0175$ whatever direction the vehicle-axis at the instant when D_1 passes through the centre-line of the track may have relatively to the centre-line of track.

It further follows that when $\epsilon = 11{\cdot}2^{mm}$, in the exceptional case when $\tan\gamma_{\text{II}} = -\frac{\sigma}{2d}$, or $\gamma_{\text{II}} = -8'\, 26''$, then the wave-height $2\gamma_{\max} = \sigma = {\cdot}0175$: for all other possible values of γ_{II} between $-\,8'\, 26''$ and $+\,8'\, 26''$, it is greater than $\sigma = {\cdot}0175$. Consequently, for $\epsilon = 11:2^{mm}$ an impact of the Fore-wheel would occur in any other position of the vehicle relatively to the centre-line of track.

If for $d = 4^{mm}$ and $\sigma = \cdot 0175$, ϵ is $> 1\cdot 7^{mm}$, then the F-wheels can run against their rails; and they will do so the more violently the greater ϵ is. The vehicle will run, consequently, more unsteadily the more ϵ exceeds the value $1\cdot 7^{mm}$.

It has been assumed that the axles can displace themselves through the angle γ_1. This is not absolutely correct, since each angular displacement is opposed by a resistance depending on the rigidity of the connexion of the bearings with the vehicle's springs.

The points D_1, D_2 cannot, therefore, describe the curves represented by the Eqn. 68. On the contrary, they must move in paths whose wave-length is greater than l, so long as an impact of the wheels does not take place.

These considerations show, consequently, that the axle-centres D_1 and D_2 move neither in the curves given by Eqn. 68, nor in those given by Eqn. 65, but really describe curves of a kind lying between these; that they approach in character more to the former when ϵ is large, and when the connexion of the axle-boxes with the vehicle's springs opposes a small resistance to an angular displacement of the axle: and to the latter when ϵ is small, and there is a greater resistance to axle angular-displacement.

The height of the wave really described by D_1 will therefore not be constant as indicated by Eqn. 68, but will increase with the length of the path described, although not to degree indicated by Eqn. 65.

According to the expressions obtained for L and l, L increases with n and d, and l with n. This shows that the length of the waves described by the middle of the axle increases with n and d: and that the vehicle will run the more steadily and smoothly the larger n and d are. Further, it follows that the length of the waves decreases as ϵ increases.

Accordingly, smooth travel of vehicles is promoted by moderate conicity of treads, slight displaceability of the axles, and a long wheel-base.*

As a consequence of the serpentine motion of the two axles the whole vehicle oscillates in the horizontal plane. The axis of this oscillation usually lies between the F. and H. A. But it can and may happen that notwithstanding the serpentine movement of the axles, the axis of the vehicle remains parallel to that of the track and that all points in the vehicle have the same uniform lateral movement. The magnitude of this swing is made up of the height of the wave described by the axles, of the amount of the clearance by which the axle-boxes can move in the axle-box guides or jaws, and of the clearance with which the axle-bushes are displaceable longitudinally in the axle-journals.

In well-maintained Passenger vehicles these clearances amount to 1^{mm} or 2^{mm} and 2^{mm} to 5^{mm}, respectively, or to a total of 3^{mm} to 7^{mm}; and generally, the smaller of these clearances are to be found when the vehicle leaves the shops with normally turned-up treads, and the larger, when the treads are worn. In the worst cases, when the swinging of the axles results from enlargement by wear of the original clearance, the lateral movement of the car-body may increase from $(3+10) = 13^{mm}$ up to $(7+25) = 32^{mm}$.

If the vehicle pendulums about a vertical axis then the lateral swing or deviation above the axles may amount to 13^{mm} or 32^{mm}; and at the ends of the car-frame it may be correspondingly more.

The frequency or number per sec. of these oscillations is approximately $\frac{l}{2V}$; or with speeds of 60—80 km./hr. it is about 2 to 3 per second.

* On the Lehigh Valley Railroad experiments were carried out as to the influence on the motion of vehicles in the straight of cylindric and conical treads respectively. The conical treads had a conicity of 1/8″. These experiments showed that vehicles with cylindric treads ran more smoothly than those with conical. The velocity in the experiments was 10 to 15 miles per hour—*Railroad Gazette*: 1882, p. 787.

To determine the influence of the clearances ϵ and σ on the steady travel of vehicles in the straight experiments on a large scale were undertaken on the Berlin-Anhalt Railway, and the results published in detail in the "Organ," 1880, p.189. These experiments showed that the vehicles given in the following Table ran smoothly in the fastest trains, when ϵ and σ had the values shown therein.

Vehicle.	d m	ϵ mm	σ mm	$\gamma_{I} + \gamma_{II}$	Remarks.
No. 372 } 369 }	4·80	2	10	7′ 6″	2-axled
217	4·08	3	10	8′ 30″	"
191	4·08	2	10	7′ 44″	"
172	4·00	1·5 to 2	10	7′ 23″	"
367	4·40	1·5	Tire in good condition.	...	"
356	6·44	2	do.	...	3-axled

In the above there was a longitudinal clearance of the axle-boxes in the axle-horns of 1^{mm}, and a clearance of the journal in the bushes of $1{\cdot}5^{mm}$.

The values of $\gamma_{I} + \gamma_{II}$ in the above Table are calculated in the manner already described from ϵ, σ, and d.

These values are somewhat greater than they would be if the axle-boxes were perfectly free to move by the amount ϵ. Thus Eqn. 69 gives, for $\sigma = 10,^{mm}$

$$\tan(\gamma_{I} + \gamma_{II}) = \cdot 005 \sqrt{\frac{2}{20.\ \cdot 5.\ 1{\cdot}5}},$$

whence

$$\gamma_{I} + \gamma_{II} = 6'\ 17.''$$

If under special circumstances when running in the *straight* both axles are displaced as far as the clearance ϵ permits from the normal middle-position into the opposite one, then the vehicle moves for the moment about the point of intersection S of **Fig. 45**, of which the distance from the track, when $\epsilon = 2^{mm}$ and $d = 4^{m}$, is given by

$$1 : \frac{1956}{2} = \frac{d}{2} : R\ (= 1956^{m}).$$

A smaller value of R is not possible; and this R is therefore the minimum radius occurring in the path of the vehicle.

But when the vehicle is provided with any device for bringing about a radialization of the axles in curves this minimum becomes very much smaller—supposing such device permits the axles to make the necessary deviation required in the sharpest curves, and that the moment, Ks, bringing about the radial position of the axles, has only to overcome the inherent frictional resistance of the mechanism of the arrangement, and not some inserted elastic resistance in addition thereto.

If the F. A. of such a vehicle having a radializing mechanism runs against the rail so that the impinging wheel mounts on the rail and so temporarily runs on a rolling circle of which the radius is, say, 5^{mm}, greater than that of the opposite wheel—which may very easily happen with the normal shape of wheel-tread—then the moment, Ks, at this axle will move the axle in an arc of a circle of radius $\rho - \cdot 5\frac{1500}{5} = 150^{m}$ until the point of intersection, S, lies at the minimum distance possible from the track corresponding to the axle's angular displacement.

If, for example, a vehicle is provided with a mechanism which will radialize axles to a curve of 180^{m} radius then the above difference of radii of 5^{mm} need only exist on the track for a distance or length of say, $\frac{5}{12}$ d, or where $d = 4^{m}$, for a distance of $1\frac{2m}{3}$, in order that S shall be 180^{m} distance from track.

Thus with such a radializing mechanism the minimum R in the path of a vehicle might be reduced from 1956^{m} to 180^{m}

These considerations lead to the conclusion that a vehicle with a radializing mechanism moves in sharp curves in consequence of the facile displaceability of the axles precisely as one does without such a contrivance: whence it follows that radializing contrivances promote unsteady motion of vehicles in the straight.

By the insertion of an elastic resistance in the radializing mechanism which shall oppose the angular displacement of the axles and shall restore the displaced axle to its normal middle-position, this disadvantageous action may be lessened or wholly removed. However, when deprived of its movability this device loses concurrently some of the advantages it offers for the passage of curves. But as soon as a "dead" travel or motion, which is to be expected through the wear due to the great stresses acting, appears in the mechanism, there occurs a too easy displaceability of the axles and a too large deviation-angle in the straight in spite of the inserted elastic resistance.

---

§ 31.

The Abrasion of Rails.

Steel rails have now for years been exclusively employed in the construction of all new lines: and, as the observations made on the diminution of their cross-sections show, are gradually worn down by the loads passing over them in proportion to the mechanical work done in friction between wheel and rail.

Consequently, the comparative duration of steel rails of the same cross-section, same quality of metal, and same degree of hardness, in any two different sections of a line is determinable in advance, if it is known in what proportion the quantities of work done in the wearing down of rails in these particular sections of the line stand to one another—assuming that the type and maintenance of the permanent-way are the same in both cases, and that approximately, at least, the same type of stock passes over the line-sections in question.

The abrasion of rails is thus dependent on the nature and the amount of traffic; and also, on whether the sections of line are level or inclined, straight or curved; and whether brakes are used thereon or not.

The observations made on the various Railways in membership with the Union of German Railway Administrations enable the wear of steel rail-heads to be estimated per gross travelling load of one million tonnes as follows:—

(1) In flat country, where there are but moderate grades (below $\frac{1}{180}$) and curves of large radii and on which brakes are not used, the abrasion amounts to from 5^{mm} to 10^{mm}.

(2) In sections with average inclines (of $\frac{1}{150}$ to $\frac{1}{120}$) and flat curves, and on which brakes are in partial use: the abrasion = 14 to 17^{mm}.

(3) In sections with inclines of $\frac{1}{100}$ to $\frac{1}{60}$ and having curves of 500^m R.; abrasion = 25^{mm}.

(4) In mountain divisions with grades of $\frac{1}{60}$ to $\frac{1}{40}$ and curves of 200^m R; abrasion = 50 to 100^{mm}.

(See "*Organ für die Fortschritte des Eisenbahnwesens*", 1886, p. 223).

As examples of severe abrasion the following observations on rail-wear may be instanced.

Railway.	Observed annual Rail head wear in millimetres.	Characteristics of the section under observation.
Atlantic and Great Western (America)	1·8	Outer-rail on a curve of 128^m R, and a grade of 1 : 83.
	1·06	Inner rail of same curve.
	·34	Gradient 1 : 151.
	·30	Gradient 1 : 1053.
Rhenish Railway	·2·5	Incline at Aachen.
Brunswick Railway	·572	
Royal Würtemburg	·35	Descent 1 : 45.
Paris-Lyons Mediterranean	·16	
Saarbrück Railway	·63 to 82	Brake-section, Dudweiler Stn.

In straights, the flanges come but rarely into contact with the sides of the rail; and consequently *most of the wear is to be found on the top or table.* The worn surface assumes a domed shape, because the rails under the influence of the rolling wheels pendulum about a longitudinal axis. This longitudinal axis is situate in the foot of the rail in wooden cross-sleeper track: and, with iron cross—or longitudinal—sleepers, at a greater or smaller distance under the same, depending on the variability of the point of support of the rail. In parts of a line where brakes are in use the rounding of the rail-head from wear is less than that on sections in which the brake is never applied, because braked wheels exercise less lateral pressure, and the rails in consequence pendulum through a smaller angle.

Rails are usually rolled to a *flatter* shape of head than that which they finally acquire through wear. However, there are makers who give the rail-head a curvature having the height of the rail as radius. All such peculiarities have, in the main, only the most insignificant influence on the wear of rails and wheels; the original curvature disappearing after a comparatively small fraction of the whole life of the rail has elapsed.

For the same reason, the frequently-proposed absolutely flat rail-top has no practical value.

The top-surface of the heads of both rails are, in the straight, equally worn, whereas the wear in curves owing to the action of the individual wheels (minutely explained in § 19) is different.

According to § 24, some 80 % of the whole curve-resistance of a 4-wheel vehicle in *sharp* curves occurs at the flange of the F-wheel — assuming that a normal contact and action of wheel and rail takes place. When this is not the case, if, for example, in a curve of 600^m R. the F-wheel flange slides on a surface of the rail-head inclined at an angle $\alpha_2 = 60^\circ$ to the axis of the wheel the resistance to be overcome at the flange of the leading wheel increases, in this instance, to $1{\cdot}4 \times 80$ %, or 112 % of the normal curve-resistance.

In Locomotives almost exactly the same thing holds: whence it follows, that *the wear due to curve-resistance shows itself principally on the inner-side of the head of the outer rail;* and that *the wear of the top-surface of rail-head both of the inner and the outer rail is, compared with the wear of the inner-side of the rail-head, quite insignificant.*

How very much more severe in sharp curves the abrasion on the inner-side of outer rail-head is than that on the top is seen from the cross-sections of rails shown in (**Figs. 46**), (**46***a*). Those of Fig. 46 are taken from a curve on the Berlin City and Suburban Railway, the latter from curves of the severely inclined and sharply-curved track on the Annaberg-Flöha Railway.

The tank Locomotives working on the Berlin City and Suburban Railway have leading-axles with wheels of $1{\cdot}12^m$ diam, and 2 coupled driving-axles having wheels of $1{\cdot}72^m$ diam.

When these engines run backwards (the driving-axles then in front), the leading driving-wheel engages—even when there is only a small previous wear of the outer-rail—with the rail-head in the conical part of its flange, and thus produces the guiding surface, only slightly domed and much inclined to the vertical, of the worn rail-head, of Fig. 46. The difference in the profiles of these rails taken from the same curve is due to the circumstance that the rails are laid on the Haarmann System—longitudinal sleepers—and are attached to each other by cross or tie-rods. (1)

It will be further seen from the cross-sections in Fig. 46*a* (2) that the flanges have completely impressed their form on the rail-head. If these profiles are compared with the other two, worn under the totally different conditions obtaining on the Berlin City and Suburban Railway, we see that all four profiles between *a* and *b* have the same shape in common.

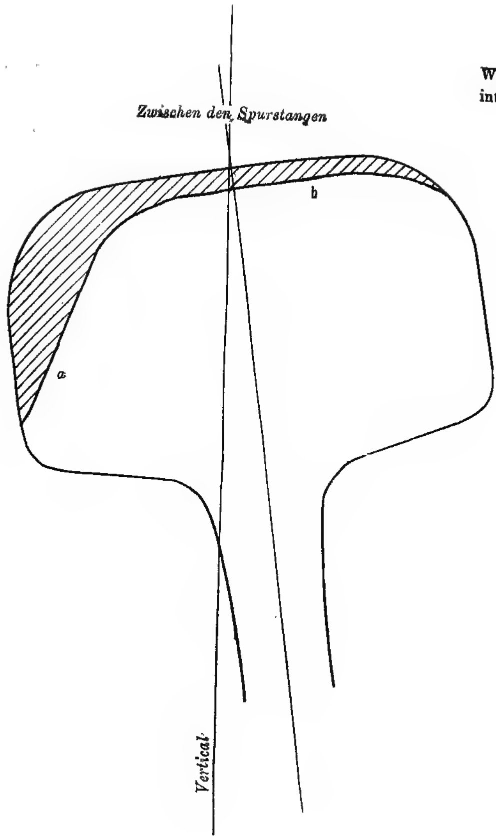
(1) The Berlin City and Suburban Railway is laid with longitudinal sleepers on the Haarmann System, with cross-tie rods in curves. [In Fig. 46 the left hand cut represents the profile of wear of the rail lying *between* the tie-rods; that on the right hand, the wear-profile *at* the tie-rods.—TRANS.]

(2) These cross sections were adopted about 14 years ago: the rails are steel-headed.

FINIS.

Fig. 46.

Wear of the Outer Rail in a curve of 300 m radius in the interval between 7th February 1882 and 9th August 1883 by the passage of 816900 axles.



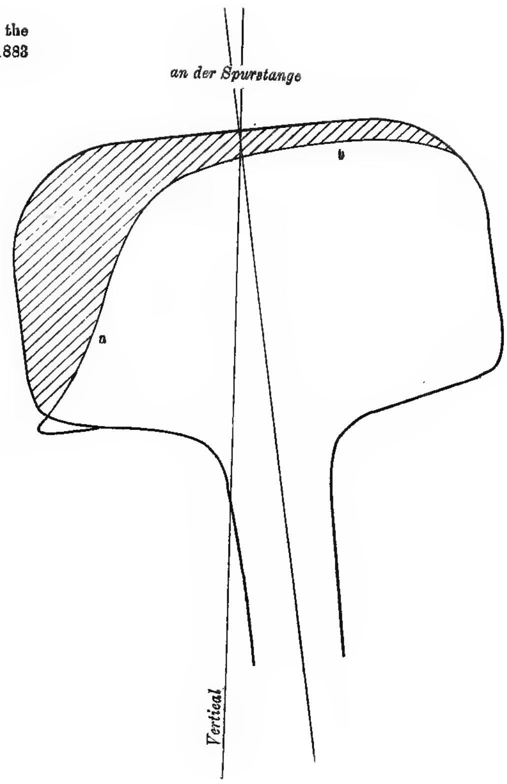
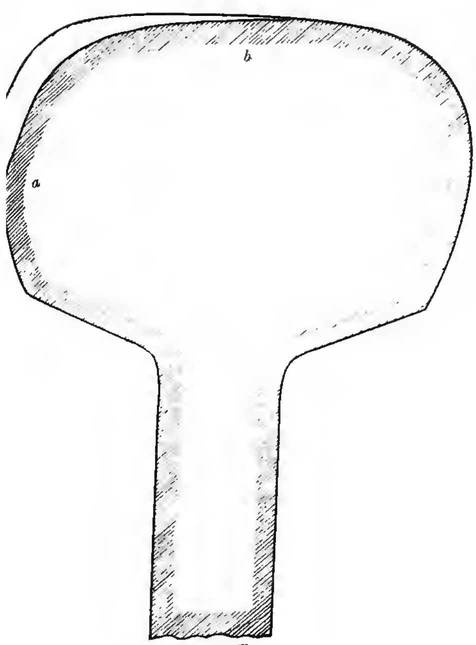


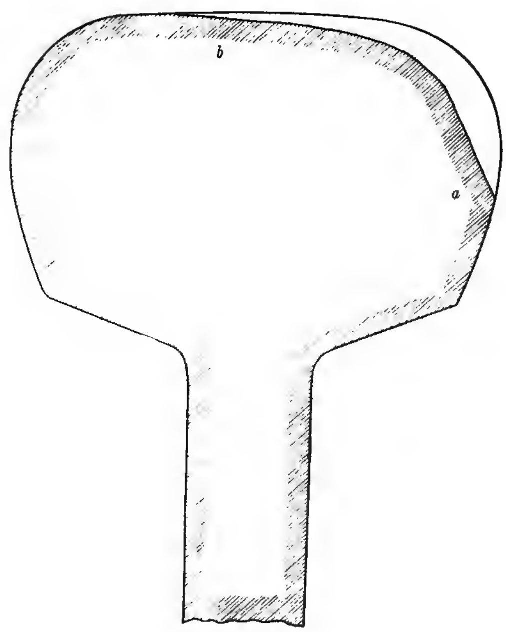
Fig. 46 a.

Wear in a single track by 9025 trains in both directions in 2¼ years



R = 169,91 m

1:100



R = 169,91 m

1: ∞

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Rad und Schiene

UND

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Nach eigener Theorie

aus der Construction der Fahrzeuge und mit Rücksicht auf die Lage des Gleises ermittelt.

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